PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 1 APRIL 15, 1915 Number 4

PHOTOGRAPHIC DETERMINATION OF STELLAR PARALLAXES WITH THE 60-INCH REFLECTOR

By Adriaan van Maanen
MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON
Presented to the Academy, March 4, 1915

The determination of stellar parallaxes had been attempted from the days of Tycho Brahe down, but without success until Bessel in 1838 succeeded in demonstrating and measuring the parallax of 61 Cygni. Since that time the work has been advancing steadily, first with the use of the meridian-circle, heliometer, and micrometer, until in 1886 Pritchard applied photography with great success. Although several astronomers have given much time to the problem, results are still comparatively few. In 1910 Kapteyn and Weersma published a list of well-determined parallaxes, which, although nearly complete, contains only 365 stars. The accuracy of these parallaxes varies considerably; their probable errors are anywhere between 0".004 and 0".151, while the mean probable error is 0".032. Since then a few lists of parallax determinations have been published, which show a good improvement; the list of Slocum and Mitchell (14 stars) has a mean probable error of 0".011; that of Miller (8 stars) of 0".011.

The material published so far cannot, however, help us very much in forming an idea of the distribution of the stars in space, as it is very one-sided. Most of the stars were chosen on account of their brightness or their large proper motions; but for the distribution of stars in space, it is clear, that we need as well the parallaxes of stars which are not supposed to be our nearest neighbors. But here arises a new difficulty; the quantities sought become so small, that only the greatest accuracy in their determination can give useful results. According to Kapteyn the mean parallax of a star of, for instance, the sixth magnitude and a

proper motion of 0".100 annually is only 0".018. It is clear that in such cases the probable error must not exceed a few thousandths of a second of arc if the results are to be valuable.

In *Contributions from the Mount Wilson Solar Observatory*, No. 79, Adams and Kohlschütter have given the absolute motions and absolute magnitudes of 100 stars with known parallaxes, for which they have determined the radial velocities. Nearly all these stars have large proper motion and are between the spectral types F and M. In the hope of completing this list by the addition of stars of the same spectral type, but of smaller proper motion, an investigation has been made to determine the accuracy with which parallaxes can be obtained with the 60-inch reflector. Although in general the method employed was that used by Schlesinger and Slocum with the 40-inch Yerkes refractor, the following deviations from their procedure may be mentioned:

The equivalent focal-length of the Cassegrain combination of mirrors employed is 80 feet, thus increasing the scale of the plates by one-quarter.

The exposure time of 15 minutes gives stars as faint as magnitude 13; by avoiding the very faintest objects visible on the plates, we can still use stars between magnitude 9.5 and 12 for comparison purposes. The fainter stars must as a rule have smaller parallaxes, so that we have here the double advantage of using comparison stars with smaller parallaxes and with a smaller mean distance from the central star.

The plates used are Seed 23, which have a fine grain; the star images therefore are sharper and the grain is less troublesome in the measuring.

The plates are measured with the blink-arrangement of the stereocomparator; two plates are compared directly without the use of any scale such as in ordinary measuring instruments. That the stereocomparator can, with some precautions, be used successfully for this kind of work was proved in a previous article (*Astronomical Journal*, 27, 140; 1912).

Special care has been taken to make the hour angles of the two plates to be compared nearly the same. The difference in hour angle has in no case been larger than 5°, while in most cases it was less than 2°.

In comparing two plates directly we avoid in the final solution for the parallax the use of a third unknown quantity; this increases the weight of the resulting parallax.

Finally, care has been taken to make the weight of the parallaxes resulting from a certain number of plates as large as possible by distributing the exposures as symmetrically as practicable within the year.

There is, however, a difficulty sometimes affecting the use of the 60-inch reflector for this work. After a rapid change in temperature dur-

ing the time when the dome is open, the mirror occasionally shows astigmatism. Under such conditions the images are more or less asymmetrical and the measures may not be as accurate as usual. This happened with three of the exposures on the last star given in the table below.

The result of the investigation has been rather promising; 5 stars have been finished, the results of which are given in the table.

Name of Star		π	*P.E.*	*Number of Exposures*
Boss P. G. C.	96	$+0''.026$	$0''.007$	14
	672	$-0\ .009$	$0\ .004$[5]	14
	1549	$+0\ .001$	$0\ .001$[5]	16
	2921	$+0\ .078$	$0\ .006$	10
	3233	$+0\ .003$	$0\ .010$	12

For the mean of 13 exposures the mean probable error of a parallax is not quite $0''.006$. If we compare this result with the best known photographic determinations of parallaxes listed below, it will be seen that we have gained considerably.

Instrument	*Observer*	*Mean P.E.*	*Number of Exposures*
Helsingfors (11-inch)[1]	Donner, Kapteyn, de Sitter	$0''.023$	12
Bonn (11-inch)[2]	Küstner, Kapteyn, de Sitter	$0''.020$	12
Cambridge, England (12.5-inch)[3]	Russell	$0''.028$	27
Yerkes (40-inch)[4]	Schlesinger	$0''.013$	37
Yerkes (40-inch)[5]	Slocum, Mitchell	$0''.011$	21
Yerkes (40-inch)[6]	Slocum, Mitchell	$0''.009$	28
Swarthmore (24-inch)[7]	Miller	$0''.011$	Not published

[1] Groningen, *Pub. Astr Lab.*, 20, 28; 1908.
[2] *Ibid.*, 23, 56; 1909.
[3] *Pub. Carnegie Inst Wash.*, 147, 65; 1911.
[4] *Astrophys. J.*, 34, 27; 1911.
[5] *Astrophys. J.* 38, 25; 1913.
[6] *Pub. Astr. Astrophys. Soc. Amer.*, 16th Meeting, 19; 1914.
[7] *Ibid.*, 17th Meeting, 1914.

The material is of course insufficient to yield any information as to possible systematic errors, but we have one evidence that these can not be very large; the mean parallax is $+0''.020$, while according to Kapteyn's table for stars of this type, magnitude, and proper motion it is $+0''.014$.

ON CHONDROSAMINE

By P. A. Levene and F. B. La Forge

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, February 8, 1915

Theoretically there are possible as many α-amino hexoses as there exist hexoses, namely, eight in the *d*-series, and as many in the *l* and *dl*-series. Of the eight possible amino-sugars only one has been discovered in nature, namely, glucosamine; this also was prepared synthetically by Fischer and Leuchs,[1] and to it the configuration of glucose was ascribed. More recent investigations have shown that two explanations for the configuration of the sugar are possible: one of glucosamine and the other of mannosamine, and that at present it is not possible to give preference to either one of the two.[2]

In the course of our investigations on the structure of chondroitin sulphuric acid we isolated an α-amino hexose isomeric with the one previously described. It was named chondrosamine.[3] On the basis of further investigation the conclusion was reached that chondrosamine has the configuration of either α-altrosamine or of α-allosamine. The grounds for the conclusion are the following:

1. On oxidation with bromine the amino-hexose was transformed into a tetroxyaminocaproic acid having $[\alpha]_D^{20.4} = -29.23$ (glucosaminic acid has $[\alpha]_D = -14.65°$). On reduction with hydroiodic acid a hydroxyaminocaproic acid was obtained. The properties of this were identical with the acid obtained on reduction of glucosaminic acid. By this was demonstrated that chondrosamine contained a normal carbon chain.

2. On oxidation with nitric acid of the deaminized chondrosaminic acid an anhydrodicarboxylic acid (epichondrosic acid) was obtained. M.P. = 201–202°C., optically inactive. C = 37.58%; H = 4.42% (theory, C = 37.50; H = 4.20).

3. On oxidation with nitric acid directly of the deaminized aminosugar an acid (chondrosic) was obtained, identical in composition: C = 37.11%; H = 4.15%; but different in properties: M. P. = 179–181°; $[\alpha]_D^{20} = -16.56°$.

4. On heating in a sealed tube at 150° with equal parts of hydrochloric and hydrobromic acid, chondrosic acid was converted into dehydromucic acid: C = 45.84%; H = 3.08% (theory, C = 46.16, H = 2.57).

5. On heating at 200°C., chondrosic acid was converted into pyromucic acid. C = 53.56%; H = 3.57 (theory, C = 53.37, H = 4.14). M.P. = 135°C. Isosaccharic acid is converted into dehydromucic

and into pyromucic acids under the same conditions as chondrosic acid, hence it was concluded that the two acids were similar in their structure, namely, that in both an oxygen bridge existed between the α- and α_1-carbon atoms.[4]

6. The anhydrodicarboxylic acids (derived from hexoseamines) have as many asymmetric carbon atoms in their molecule as the corresponding acids derived from hexoses. Therefore, there are possible only two optically inactive anhydrodicarboxylic acids: namely, one corresponding to mucic and the other to allomucic acid. This consideration limits the possibilities of configuration of epichondrosic acid to anhydromucic, or anhydroallomucic. The fact that chondrosamine forms a phenyl osazone identical in its properties with that of allose decides the choice between the two configurations in favor of anhydroallomucic.

7. Regarding the configuration of chondrosine the choice remains between that of *l*-allosamine or of *l*-altrosamine. Both anhydroallomucic and anhydrotalomucic acids are obtainable from chondrosamine depending on the procedure in preparation.

It was attempted to prepare chondrosaminic acid synthetically from ribosimine by the action of prussic acid. The acid obtained in this manner had the composition of hexosaminic acid, C = 37.02%, H = 6.58%, and N = 7.44% (theory, C = 36.92, H = 6.66, N = 7.18). M. P. = 198°C., $[\alpha]_D^{28} = -9.4°$. The acid was evidently epimeric with chondrosaminic, and on treatment with nitric acid it should have yielded chondrosic acid. Unfortunately for lack of material this experiment had to be deferred.

[1] Fischer, E., and Leuchs, H., *Berlin, Ber. D. chem. Ges.*, 36, 24 (1903).
[2] Irvine, J. C., and Hynd, A., *London, J. Chem. Soc.*, 101, 1128 (1912); 105, 698 (1914).
[3] Levene, P. A., and La Forge, F. B., *J. Biol. Chem.*, 18, 123 (1914).
[4] Fischer, E., and Tiemann, F., *Berlin, Ber. D. chem. Ges.*, 24, 2139 (1891).

THE FREEZING-POINT-SOLUBILITY LAW FOR IDEAL SOLUTIONS

By Edward W. Washburn and John W. Read

LABORATORY OF PHYSICAL CHEMISTRY, UNIVERSITY OF ILLINOIS

Presented to the Academy, February 19, 1915

When a solution is formed by mixing together two completely miscible pure liquids, it is found that when the two constituents resemble each other very closely the formation of the solution takes place without appreciable heat-effect or volume-change. Such solutions have been termed *ideal solutions;* and experiments have shown that the vapor pres-

sure p_A of any constituent A in such a solution is proportional to the mole-fraction x_A of that constituent in the solution. This law is expressed in differential form by the equation: $dp_A = k dx_A$.

From this relation, with the aid of thermodynamics, we may derive the other laws of ideal solutions. Most of the experimental data at present available for illustrating the application of these laws to specific cases have been derived from direct measurements of the vapor-pressures themselves. It seemed therefore desirable to secure some additional experimental data which would serve as examples illustrating the character of the results given by some of the other laws.

The freezing-point-solubility law was chosen as the first one to be studied in this connection because it happens that this law offers a convenient as well as exact method for testing the accuracy with which a given solution obeys the laws of ideal solutions.

For an ideal solution of two substances, A and B, it can be shown thermodynamically that the following equations express the change of freezing-point caused by an increase in the mole-fraction of each component:

$$\frac{dT}{dx_A} = \frac{RT^2}{L_A x_A}; \quad \frac{dT}{dx_B} = \frac{RT^2}{L_B x_B}$$

where R is the gas constant, T the absolute temperature, L the heat of fusion per mole, and x the mole-fraction, in solution, of the substance indicated by the subscript. At the eutectic point it is evident that these two equations must hold simultaneously; and by eliminating x_A and x_B from them we can calculate the eutectic temperature for the system under consideration. The purpose of this investigation was to test in this way the validity of these equations for the three two-component solutions which can be formed from the three substances benzene, diphenyl, and naphthalene. For this purpose it is necessary to know the heat of fusion and melting-point of each substance.

Satisfactory values of the molar heats of fusion of benzene and naphthalene were available; that of benzene had been determined by Demerliac and by J. Meyer to be 2370 calories, and that of naphthalene by Alluard and by Pickering as 4560 calories. It was, however, necessary to determine that of diphenyl. For this purpose, weighed amounts (about 30 grams) of diphenyl in stoppered test-tubes were transferred from a mercury bath at 71° to a calorimeter, consisting of a silvered vacuum-tube filled with water; and the rise in temperature (about 3°) was measured with a Beckmann thermometer. The calorimeter was then cooled to its original temperature by dropping in a weighed piece of dry

ice taken from an air bath at $-2°$ where it was suspended on a silk thread. As an additional check on the water-equivalent of the calorimeter the first operation was then repeated, using however test-tubes containing naphthalene instead of diphenyl. The heats of fusion of ice and of naphthalene being known, that of diphenyl could be readily calculated from these experiments. It was thus found to be 4020 calories per mole.

The freezing-points of the pure substances and the eutectic points of the three solutions were determined by the equilibrium method. The apparatus consisted of a Baudin thermometer standardized by the National Bureau of Standards and a small silvered cylindrical vacuum-tube which was heated (or cooled) to the required temperature previous to beginning the experiment. For temperatures higher than that of the room, the crystals (pure substance or eutectic mixture) were melted in a small casserole, and the liquid (about 25 cc.) was poured into the vacuum-tube and the thermometer inserted. When the temperature of crystallization had been reached more of the crystals were added, after which the thermometer reading soon became constant and remained so (within less than 0.1°) for ten or fifteen minutes, with constant stirring and tapping of the thermometer. For temperatures lower than room-temperature the solid crystals (previously cooled, in the case of the eutectic mixture) were placed in the vacuum-tube, and equilibrium between the crystals and liquid in contact with the thermometer bulb was obtained as before. In this way the three melting-points and the three eutectic temperatures could be easily measured with an accuracy of at least 0.1°. The constancy of the three melting-points within this limit is sufficient criterion of the purity of the materials employed.

In order to calculate the eutectic temperatures from the freezing-point equations given above, these equations were first integrated on the assumption that L is independent of T for the ranges involved. This assumption is justified in the case of benzene and naphthalene by the small differences between the specific heats in the solid and liquid states. In the case of diphenyl specific-heat data are not available, but the assumption seemed justifiable by analogy. The two integrals are:

$$2.303 \log_{10} x_A = -\frac{L_A(T_{0A} - T)}{RT_{0A}T}; \quad 2.303 \log_{10} x_B = -\frac{L_B(T_{0B} - T)}{RT_{0B}T};$$

where T_{0A} and T_{0B} are the absolute melting-points of the pure substances. By solving these two equations simultaneously (with the aid of the axiom, $x_A + x_B = 1$) values of the eutectic temperature T were obtained.

The melting-points of the pure substances were found to be: Benzene,

C_6H_6: 5.48°. Diphenyl, $C_6H_5C_6H_5$: 68.95°. Naphthalene, $C_{10}H_8$: 80.09°. The directly measured values and the computed values of the three eutectic temperatures were found to be as follows:

Benzene-naphthalene; measured: −3.48°; computed: −3.56°.
Benzene-diphenyl; measured: −5.8°; computed: −6.1°.
Naphthalene-diphenyl; measured: 39.4°; computed: 39.4°.

The close agreement between the calculated and observed values of the eutectic temperatures shows that these three solutions are very close to ideal solutions in their behavior; for this method of testing them is a very rigorous one.

The above given differential equations represent the most generally applicable freezing-point and solubility law which we have at present. By means of this law we can calculate the freezing-point of a given solution or the solubility of a given solid in a liquid for all solutions which fulfil the necessary conditions. These conditions are (1) the heat of dilution of the solution for the range of concentration involved must be practically zero; and (2) the dilution of the solution must not be accompanied by the occurrence of a chemical reaction of any kind. The quantity L in the equations represents in general the molal heat of solution of the crystalline substance in its own saturated solution. For ideal solutions this heat of solution is of course identical with the heat of fusion. The above law should be generally employed in place of the more common 'dilute solution' laws, which are only special cases of it.

In closing this paper it may be worth while to point out two examples of the use of the equation given above which do not seem to be familiar to most chemists, and especially to those chemists in whose special fields of work they are of chief interest.

1. The results of the search for some general principle concerning the solubility of substances in each other is usually summed up by restating the old principle that *similia similibus solvuntur*. In the light of our present knowledge we can, however, go somewhat further than this simple qualitative statement, and can formulate the following quantitative principle:

When a crystalline substance A is dissolved in any solvent B with which it forms an ideal solution, its solubility (expressed in terms of its mole-fraction in the saturated solution) is *entirely independent of the nature of the solvent B*, and is therefore the same in all such solvents. The solubility depends merely upon the temperature and upon the melting-point and heat of fusion of the crystals A, from which data it can in fact be readily calculated. This important principle, which follows directly from the above equations does not seem to have been gener-

ally appreciated, although it was discovered in 1893 by Schröder (*Zs. physik. Chem.*, 11, 449).

2. Another important use to which the general freezing-point–solubility law could be put would be in settling the frequently discussed question as to whether racemates exist as such in the liquid state, and if so, to what extent. This question could be most certainly and definitely decided for any given pair of optical isomers simply by determining the freezing-point diagram for the system and applying the general law. The exact amount of racemate present in the liquid state and the manner in which it varies with the temperature could also be computed; for a solution composed of two optical isomers forms one of the most perfect examples known of an ideal solution, and the laws of such solutions would apply with great exactness over the whole range of concentrations. In addition to the freezing-point diagram, the latent heats of fusion and the heat of dilution would also be needed in order to work out the problem completely. The main point which it is desired to emphasize here is simply that the ideal solution laws are capable of giving a complete and quantitative answer to such a question as this, an answer concerning the correctness of which no reasonable doubt could be felt.

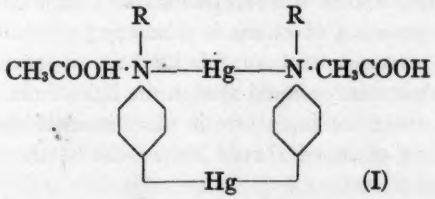
MERCURY DERIVATIVES OF AROMATIC AMINES. I. STRUCTURE OF PRIMARY AND SECONDARY *p*-AMINOPHENYLMERCURIC COMPOUNDS

By Walter A. Jacobs and Michael Heidelberger

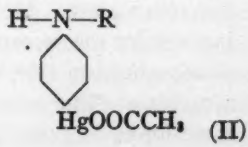
ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, February 17, 1915

During the year 1902, the question of the structure of the mercury derivatives of the aromatic amines was the occasion of a polemic between L. Pesci[1] and O. Dimroth.[2] The former ascribed the complex structure (I) to the mercuriated primary and secondary amines (R = H or alkyl), while Dimroth advocated the simple monomolecular formula (II), basing his views upon arguments which Pesci was unable to controvert. Subsequent workers have used the monomolecular formula



(I)



(II)

indicated by Dimroth's reasoning, although no evidence of a direct nature was adduced and only the reactions of the mercury were considered in assigning the formula. It seemed to the writers that a study of the amino group in these compounds would furnish more direct evidence as to the structure than any at hand, for, obviously, a compound of the formula (I), where R = H, could not undergo the reactions characteristic of the primary amino group. This work was therefore undertaken as a preliminary phase of a more extended study of the mercuriated aromatic amines. It was found possible to prove formula (II) for *p*-aminophenylmercuric acetate by demonstrating the presence of the primary amino group in two ways: (1) the formation of a salicylidene derivative; (2) the ready formation of a diazo solution, from which were prepared the azo dyes with dimethylaniline, diethylaniline, phenol, resorcin, and α-naphthylamine-5-sulfonic acid. It was also found possible to prove the monomolecular structure of *p*-methylaminophenylmercuric acetate by demonstrating the presence of a secondary amino group through the preparation of the nitroso derivative. A more detailed discussion and a description of the compounds prepared will appear in the *Journal of Biological Chemistry*.

[1] *Zs. anorg. Chem.*, **32**, 227 (1902).

[2] *Berlin, Ber. D. hem. Ges.*, **35**, 2038 (1902); *Zs. anorg. Chem.*, **33**, 311 (1902).

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHANE DERIVATIVES. I. GENERAL DISCUSSION

By Julius Stieglitz

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Read before the Academy, December 9, 1914. Received, February 20, 1915

Introduction.—In most chemical actions taking place at moderate temperatures, a change affecting an atom or a group of atoms in a molecule leaves the remaining atoms in undisturbed relations to one another. This stability of groupings of atoms led, especially in the domain of organic chemistry, to the theory of the structure of molecules and underlies our methods for determining these structures. Occasionally, however, the study of the product of a reaction shows that a profound rearrangement of the grouping of atoms in a reacting molecule has taken place. Molecular rearrangements of this kind are of special interest for the reason that their study should shed some light, from a purely chemical side, on the forces holding atoms in place in molecular structures. The evidence thus obtained should aid in the study of these forces by purely physical means.

The recent development of electron theories of valence has given the investigation of the causes underlying molecular rearrangements an opportunity for much greater precision in formulating the problems and in expressing the results of their study. Thus, the original electron theory of valence proposed by J. J. Thomson[1] has been used with advantage in interpreting the molecular rearrangements of hydroxylamine and halogen amine derivatives of certain organic compounds independently and practically simultaneously by L. W. Jones[2] and myself.[3] These interpretations differed in some important particulars, but agreed as to the fundamental relations.

The electron theory will here be used in expressing the results of recent investigations of molecular rearrangements of triphenylmethane derivatives. Inversely, certain definite conclusions may be drawn from the study of the rearranging compounds in regard to assumptions made in the various theories of valence; and it is proposed to present this evidence first as a contribution to the greater problem of valence from the field of pure organic chemistry—the very field in which newer views are most urgently advanced by physicists and physical chemists.

The Electron Theories of Valence and the Molecular Rearrangements of Halogen Amines and Similar Derivatives. When an acyl halogen amine R.CO.NH(Hal) is treated with bases, it undergoes the Hofmann rearrangement, the radicle R (which represents an alkyl or acyl radicle or hydrogen) being torn from its union with carbon and carried to the nitrogen.[4] We have, for instance:

$$R.CO.NH(Cl) + NaOH \rightarrow NaCl + HOH + CO{:}NR. \quad (1)$$

Triphenylmethyl halogen amines, we have found,[5] undergo the same kind of rearrangement:

$$(C_6H_5)_3C.NH(Cl) + NaOH \rightarrow NaCl + HOH + (C_6H_5)_2C{:}NC_6H_5 \quad (2).$$

Now, in both these classes of chlorine derivatives, the chlorine unquestionably is present as the specific radicle Cl of hypochlorous acid:[6] for the compounds are prepared from hypochlorous acid; they form the acid by hydrolysis; they form hypochlorous esters with alcohols, hypochlorous amides with amines; in all of these reactions the specific oxidizing power of the hypochlorous acid radicle remains intact; all attempts to replace the chlorine atom by negative groups (OH, OR, NH_2) have proved futile; all attempts to prepare chloroamines by means of the hydrogen chloride radicle Cl and its derivatives (HCl, PCl_3, etc.) have likewise proved ineffectual. On the other hand, in compounds such as the alkyl chlorides $R_3C.Cl$ and acyl chlorides R.OC.Cl, the chlorine is un-

questionably present as the specific hydrogen chloride radicle Cl: the compounds are prepared from hydrogen chloride or its derivatives (PCl_3, etc.); they form hydrochloric acid by hydrolysis, by treatment with ammonia and a variety of other reagents; the chlorine can be readily replaced by a great number of negative radicles; and in these chlorides, the chlorine has no oxidizing power.

There is, evidently, a fundamental difference in the condition of the halogen in these organic compounds, the rearranging chloroamines and the (non-rearranging) alkyl and acyl chlorides. This difference is sharply and precisely expressed in terms of Thomson's original theory of electron valences by the electron structure (Cl^+) for the hypochlorous acid radicle,[7] the positive charge indicating that the atom has lost an electron, and by the structure Cl^- for the hydrochloric acid radicle, the negative charge being due to an electron captured by the atom. The chloroamines then have the structure $R_2N^{-+}Cl$, the alkyl and acyl chlorides, the structures $R_3C^{+-}Cl$ and $R.OC^{+-}Cl$. (Here and throughout these articles only the valences in question are indicated.) These electronic structures give a complete and exact history of all the striking characteristics of the two groups of compounds, as just enumerated.

According to the more recent suggestions of J. J. Thomson,[8] the valences holding atoms in molecules, especially in the case of non-electrolytes, may be formed, not only in the fashion assumed in his original theory (called now 'intramolecular ionization'), but also by the electrical attraction between the normal electrons of one atom and the 'positive core' of the other, each atom remaining, as a whole, electrically neutral. Practically the same suggestion (following one made by A. A. Noyes[9] in 1907 that we might have two kinds of molecules of one and the same compound, as, for instance, HCl and H^+Cl^- for hydrogen chloride) had previously been made by Bray and Branch[10] and by G. N. Lewis,[11] who speak both of 'non-polar' valences as uniting atoms, especially in non-electrolytes, and of 'polar' valences, of the type assumed in Thomson's original theory. Thomson defined and elaborated more precisely than did the American chemists the nature of 'non-polar' valences. Both Lewis and Thomson consider that the valences may be polar in one part, non-polar in another part of the same molecule. (The terms non-polar and polar will here be used as signifying the two kinds of valence postulated also by Thomson's later theory, inasmuch, as they are much more convenient than the terms 'intra-molecularly ionized' and 'non-ionized' valences.)

It is evident, from the development given above, that even according to these newer views of valence, the electronic structure of the chloro-

amines, as far as the chloroamino radicle is concerned, must be a *polarized* one, $R_2N^{-}{}^{+}Cl$ and not the non-polarized union $R_2N\rightleftarrows Cl$ permitted by the new theories, and that, similarly, the electronic structures of the alkyl and acyl chlorides should be considered polar, $R_3C^{+}{}^{-}Cl$ and $ROC^{+}{}^{-}Cl$. The non-polar structures $R_2N\rightleftarrows Cl$ and $R_3C\rightleftarrows Cl$ would not account at all for the persistent, fundamental differences in the properties of the chlorine in the two series of derivatives. Now, while both the chloroamines and the alkyl and acyl chlorides may be minutely ionizable, they certainly are not electrolytes in the ordinary acceptance of the term. We thus find that strongly polarized combinations of atoms need not produce electrolytes. This being the case in an instance open to demonstration, one is led inevitably to the inquiry, why it should not be true for a great many polarized combinations and whether the assumption of non-polar unions is at all necessary.

Very strong, if not so obvious, evidence, leads to the conclusion that the relations developed for the chloramines and the alkyl chlorides are duplicated by other series of compounds, for instance by the hydroxylamines[12] and the alcohols, whose relations may be expressed precisely and completely only in the polar structures $R_2N^{-}{}^{+}OH$ and $R_3C^{+}{}^{-}OH$. Again we have no exhibition of markedly basic properties by the alcohols (which form oxonium salts with acids and are 'base-forming' like ammonia, but not bases, ionizing directly simply as weak acids) in spite of the strong evidence of intramolecular ionization.

Finally, it should be said in this connection, that exactly the same persistent fundamental differences as exist between positive and negative chlorine in organic compounds are clearly manifested between nitrogen as the nitrous acid radicle N^{+++}, say in the diazo compounds[13] and nitrogen as the ammonia radicle $N\equiv$ in the amines, amides, and nitriles. Here again, the relations are precisely expressed by polar structures for the nitrogen radicles without attendant indications of any tendencies towards ionization.

Nothing stated in this article is, however, to be interpreted to the effect that one radicle may not be converted into the electromeric one of opposite sign; but all such transformations represent explicit, easily recognized oxidation-reduction reactions.

Turning now to a further phase of this problem, we find that in the molecular rearrangements expressed in equations 1 and 2, the hypochlorous acid radicle of the chloroamine is converted into the common negative chloride ion of sodium chloride. According to the original Thomson theory of valence, in this change the positive chlorine atom of the chloroamine must capture two electrons from some other atom or atoms;

and, since the reagent used remains in the same condition of oxidation throughout the reaction, these electrons must be taken from some neighboring atom or atoms in the original molecule. This, obviously, must disturb the valences holding the atoms in the molecule, and a molecular rearrangement is altogether inevitable.

The evidence as to the charges on the individual atoms and radicles is clearest for triphenylmethylchloroamines and its analogs,[3] and we must formulate the rearrangement in the following way, the charges that are changed being indicated by heavy type:

$$(C_6H_5^+)_2{}^{-}_{-}\underset{\underset{(C_6H_5^+)}{|}}{C^{+}_{-}}{}^{-}\underset{\underset{Cl^{\mathbf{+}}}{|}}{N^{\mathbf{=}}_{\mathbf{=}}}{}^{+}H \xrightarrow{-H^{+-}Cl} (C_6H_5^+)_2{}^{-}_{-}\underset{\underset{(C_6H_5^+)}{|}}{C^{+}_{\mathbf{-}}}{}^{-}N \rightarrow (C_6H_5^+)_2{}^{-}_{-}C^{+}_{\mathbf{+}}{}^{-}N^{\mathbf{=}}_{\mathbf{=}} \; (C_6H_5^+)$$

$$\rightarrow (C_6H_5^+)_2{}^{-}_{-}C^{+}_{+}{}^{-}_{-}N^{-}({}^{+}C_6H_5) \qquad (3).$$

It is assumed that Cl^+ takes the two electrons first from the nitrogen originally holding it. (The same result would be obtained if the electrons were taken directly from the carbon by the positive chlorine. Evidence on this part of the problem is presented in the following article.) The nitrogen is thus rendered incapable of holding the H^+ and Cl^- and these are lost, forming sodium chloride and water with the reagent sodium hydroxide, as indicated in equation 2. The unsaturated (univalent[14]) nitrogen, in turn, takes two electrons from the carbon atom holding it, and the new positive charge produced thereby on the carbon atom must release one of the positive phenyl groups and this must move, under the force of the electrical field, to the negative nitrogen.

We recognize that the disturbing, labile element in the original structure is just the unstable, positive chlorine, so admirably represented by Thomson's first theory, and that its undeniably great tendency to go over into its common stable negative form is ***the original source of the molecular disturbance and rearrangement.***

The product of the rearrangement, phenylimido benzophenone, in harmony with the electronic structure resulting for it according to equation 3, is uniformly hydrolyzed by acids to benzophenone and aniline:

$$(C_6H_5)_2C^{+}_{+}{}^{-}_{-}N^{-}({}^{+}C_6H_5) + HOH + HCl \rightarrow (C_6H_5)_2C^{+}_{+}{}^{-}_{-}O + (C_6H_5^{+-}N \equiv H_3^+)^{+-}Cl \qquad (4).$$

It is evident that the application of the original theory of Thomson to the valence relations in triphenylmethylchloroamine gives us a complete and precise picture of the cause, mode, and product of its molecular rearrangement.

If we attempt to express the rearrangement in terms of the theory

that we may have non-polar, as well as polar valences existing simultaneously in molecules, we find the following: (1) It is necessary to assume polar valences in the chloroamine radicle, as was proved above. (2) We recognize the positively polarized chlorine as the unstable element in the molecule. (3) In the product, phenylimido benzophenone, we must assume at least partial polarization of the methylene carbon and nitrogen, in order to express precisely its mode of hydrolysis (equation 4). (4) It is necessary to assume therefore that at least one electron has migrated from the methyl carbon to the nitrogen (or chlorine). (5) To these definite conclusions may be added the fact that a presumptively negative group ^-OR attached to carbon never migrates[15] to nitrogen under these conditions, it is only presumably positive alkyl (CH_3^+) or alphyl ($C_6H_5^+$) radicles and hydrogen radicles H^+ that migrate thus. In other words, so far as experiment penetrates into these molecular rearrangements, we find in the original molecule, in the reaction itself and in the product of the rearrangement evidences only of polar valences. The later modifications of the Thomson theory are of no assistance in formulating the theory of these reactions.

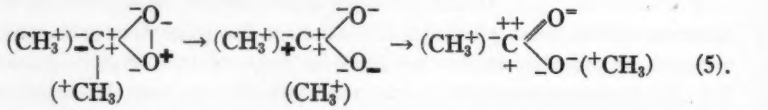
Finally, there is a whole group of rearrangements analogous to the rearrangement of chloroamines. In each instance one can plainly recognize, as the common characteristic of all the rearranging compounds, that the labile component in the original molecule is an unstable positive radicle or atom, which tends to go over into its stable negative form. In every instance, this tendency is found satisfied in the action, and as this involves the absorption of electrons from neighboring atoms in the original molecule, one must conclude that in all of these cases the tendency mentioned and its satisfaction bring about the rearrangement. These relations hold for the following groups of compounds:

1. *Hydroxylamine* derivatives, such as $(C_6H_5)_3C.HN^{-+}OH$ (considered in the following article), in which the positive hydroxide group[12] is unstable, tending to form the well known negative group. The molecular rearrangement may be expressed[12] by the same equation 3 as was used for the chloroamine, if ^+OH is substituted for Cl^+ and HO^- for Cl^-.

2. *Azides*, such as $(C_6H_5^+)_3\equiv C^{+-}N\!=\!\overset{+}{N}\!=\!\underset{+}{N}\!=\!N^{+}_{+}$ (see the following article), which tend to lose the neutral molecule N_2 with a gain of two electrons for the radicle N_2^{++}. Again equation 3 may be used to express the whole action, the loss of N_2 being substituted for the loss of HCl. (The azide is formed by the action of nitrous acid $N^{++}_{+}\!=\!O(^-OH)$ on the hydrazine $(C_6H_5)_3C.HN^{-+}NH_2$,[16] which leads to the structure used.

The same net result would be shown if the alternative structure[17] $(C_6H_5)_3C^{+-}N^{-}(^{++}N^{+++})\equiv N$ were accepted for the azide.)

3. *Peroxides*, in whose rearrangement a positive charge on an oxygen atom is converted into a stable negative charge:



(5).

In conclusion, the effects of an electric current in decomposing the salts of organic acids are entirely analogous, fundamentally, to these molecular rearrangements.[12] With the current the effect is produced by the positive electrode charge, with these compounds, by the positive polar charges on the specific atoms, as just shown.

The original Thomson theory of electron valences is evidently extremely useful in the precise formulation of these molecular rearrangements, involving a large group of organic reactions. Of the newer theories of Thomson, Bray and Branch, and Lewis, we find occasion to use only so much as is already found in the original theory. This, of course, does not exclude the possibility that the newer assumptions may reveal their usefulness in other fields of investigation. (Thomson's structure for benzene, developed on the basis of the newer theory,[18] interpreted rigorously and without further limiting assumptions, seems to make possible at least *four* di-substitution products C_6H_4XY, while only the *three* well known series have been recognized.)

(For references see end of following article.)

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHANE DERIVATIVES. II. EXPERIMENTAL PART

By Julius Stieglitz and Collaborators

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Read before the Academy, December 9, 1914. Received, February 20, 1915

MOLECULAR REARRANGEMENTS OF TRIPHENYLMETHYL HALOGEN AMINES[5]

WITH ISABELLE VOSBURGH † AND AGNES FAY MORGAN

The study of the mechanism of the molecular rearrangement of acyl halogen amines RCO.NH(Hal) is complicated by the fact that the salts formed by the bases used as reagents to start the rearrangement may

† My young collaborator, Miss Vosburgh, who carried out the first part of the experimental work of this investigation most skilfully and conscientiously, died December 4, 1914, as the result of an automobile accident.—J. S.

have the tautomeric structures, RC(OMe) (:NHal) or RCO.N(Me)Hal. This complication does not exist in the case of triphenylmethyl halogen amines $(C_6H_5)_3C.NHHal$, whose metal derivatives could only have the definite structure $(C_6H_5)_3C.N(Me)Hal$. A successful molecular rearrangement of these halogen amines by bases would give convincing evidence as to the nature of the salt formed in initiating the rearrangement and would give strong presumptive evidence in favor of the view that in the rearrangement of acyl halogen amines, the salts RCO.NMeHal are the ones directly concerned in the rearrangement.

Further: chloroimidoketones $(C_6H_5)_2C:N^{-+}Cl$ have been found to resist rearrangement[19] in spite of the presence of the unstable positive chlorine atom (see part I). This result was not unexpected from the point of view of the theory of the one of us concerning the mechanism of these molecular rearrangements:[4] the theory postulates that the easiest path to a rearrangement is through the intermediate formation of univalent nitrogen derivatives $RCO.N<$, $R_3C.N<$ (see equation 3), the free valences of the univalent nitrogen being considered sufficiently powerful to tear the radicle R from the carbon and thus effect the rearrangement to a more stable molecule (see equation 3). The significance of the intermediate formation of a univalent nitrogen derivative in relation to the electron theory is discussed later on in the text. Triphenylmethyl halogen amines, $R_3C.NH(Hal)$ and $R_3C.N(Hal)_2$, in contrast with the chloroimidoketones $R_2C:NCl$, present the opportunity for an unhampered formation of the univalent nitrogen derivative $R_3C.N<$ by the loss of H(Hal) or $(Hal)_2$ under suitable experimental conditions. They were therefore expected to be easily subject to rearrangement.

This investigation was undertaken with the object of deciding the questions indicated and thus throwing light on the whole problem of these molecular rearrangements. The experimental results confirmed in every respect the theoretical anticipations. Of the facts established and their bearing, the following are the most important:

1. A number of triphenylmethyl halogen amines—$(C_6H_5)C.NHBr$, $(ClC_6H_4)_2(C_6H_5)C.NHCl$, $(BrC_6H_4)(ClC_6H_4)(C_6H_5)C.NHCl$, and $(C_6H_5)_3C.NCl_2$ and $(ClC_6H_4)(C_6H_5)_2C.NCl_2$—were found to undergo under suitable conditions a molecular rearrangement into phenylimido benzophenones $R_2C:NR$ (see equation 3). The rearrangement was established in the case of the first compound listed by the identification of the phenylimido benzophenone $(C_6H_5)_2C:NC_6H_5$ (yellow crystals, m.p. 110°; the melting-point of a mixture of the product with synthetic substance (m.p. 111°–112°) was found at 111°) and by the identification of the benzophenone (m.p. 47°) and aniline (bleaching-powder test; the

chloroplatinate gave 32.41% platinum, theory 32.74%) produced by the hydrolysis of the imido ketone (see equation 4). In the rearrangement of the chloroamines containing substituted phenyl groups, mixtures result: in a fraction of the rearranging molecules the substituted phenyl radicle migrates to the nitrogen, in the remainder the phenyl group itself leaves the carbon for the nitrogen. Thus, in the rearrangement of di-parachlorophenyl-phenyl methylchloroamine (the second compound listed above), the two substances $(C_6H_4Cl)_2C{:}NC_6H_5$ and $(ClC_6H_4)(C_6H_5)C{:}NC_6H_4Cl$ are formed. This was established by the separation and identification of their respective products of hydrolysis (equation 4), dichlorobenzophenone $(ClC_6H_4)_2CO$ (m.p. 141°) and aniline (bleaching-powder test), and chlorobenzophenone $(ClC_6H_4)(C_6H_5)CO$ (m.p. 77°) and chloroaniline (m.p. 70°). In each instance the melting-point of a mixture of the substance obtained with the corresponding synthetic product was taken to complete the identification. The ratios in which the various radicles migrate to the nitrogen in the rearrangements was determined by quantitative examinations of the anilines formed: for the instance just discussed, the ratio of chloroaniline to aniline was found to be 68/32, in molar terms—in other words, the two radicles share in the migration very closely in the same ratio as they are found in the original molecule.

2. The reagents used to effect the rearrangements are those which the univalent nitrogen theory would postulate: bases (sodium and calcium hydroxides, sodium methylate) were used for the rearrangement of the monohalogenamines to facilitate the loss of halogen acid; and, characteristically, heat alone was sufficient to cause the loss of chlorine with explosive violence by the dichloroamines, the rearrangement ensuing:

$$(C_6H_5)_2C.NCl_2 - Cl_2 \rightarrow (C_6H_5)_2C.N < \rightarrow (C_6H_5)_2C{:}NC_6H_5 \qquad (6).$$

3. A compound, triphenylmethylchloromethylamine $(C_6H_5)_3C.N(CH_3)Cl^+$, analogous to these rearranging bodies, with the exception that the well known stability of the union between nitrogen and the methyl radicle would interfere with the formation of a univalent nitrogen derivative, was found *not to be rearranged*, either by treatment with bases or by heat. A rearrangement would have produced a derivative of methylaniline (see below), which by hydrolysis would have given methylaniline. All the tests for methylaniline, even the exceedingly sensitive chloride of lime color test, showed its absence in the products of hydrolysis of the compounds formed under the conditions mentioned.

This important result shows that the tendency of a positive halogen

atom to go over into its stable negative form is not alone sufficient, as a rule, to cause the characteristic rearrangement of these compounds, but that the path to the formation of a univalent nitrogen derivative must usually, if not always, be open, this derivative being the substance that suffers the actual molecular rearrangement (see equation 3). This must signify that a positive halogen atom can secure two electrons from the nitrogen atom to which it is directly linked much more readily than from the carbon atom holding the nitrogen (see equation 3). Otherwise the rearrangement would proceed by a direct exchange of radicles following a migration of the electrons from the carbon to the halogen atom. Furthermore, the characteristic contrast shown in the behavior of the rearranging compounds ($R_3C.NH(Hal^+)$—losing halogen acid—and $R_3C.NCl_2^+$ – losing chlorine –), on the one hand, and the non-rearranging compounds ($R_3C.N(CH_3)Cl^+$ and $R_2C{:}NCl^+$), on the other hand, leads to the important conclusion that the electrons most easily captured from the nitrogen atom by the positive halogen atom are *the two valence electrons* beyond the normal electrons of the nitrogen atom rather than any of the electrons of the normal atom. Otherwise, again, we should have a rearrangement by a direct exchange of radicles following the exchanges of electrons. This conclusion is included in equation 3 of the preceding paper.

The conclusions reached—(1) that a positive halogen atom can secure electrons more readily from the atom to which it is directly attached (nitrogen, in this case) than from one further removed from it, and (2) that the more readily available electrons are the two extra valence electrons of the nitrogen atom rather than its normal ones, seem not unreasonable. The second conclusion is in agreement with the observations of physical chemistry on the oxidation of negative ions like $S^=$, I^-, etc. This result seems to us the most important one established by the investigation, giving us a logical explanation, from the point of view of the electron theory of valence, of the real significance of the univalent nitrogen in the author's theory of the mechanism of this and similar rearrangements.

THE MOLECULAR REARRANGEMENT OF β-TRIPHENYLMETHYL β-METHYL HYDROXYLAMINE

WITH BERT ALLEN STAGNER

While a great mass of evidence, gathered by the author as well as independent investigators,[4] has shown that the molecular rearrangements of halogen amines, hydroxylamine derivatives and azides seem to proceed preferably by the path of an intermediate formation of uni-

valent nitrogen derivatives, the theoretical possibility that another path might be taken toward the formation of more stable molecules has always been kept in mind. As a consequence, in the investigation of every new type of rearrangement, experiments have been included to attempt the rearrangement of compounds which could not readily form univalent nitrogen derivatives (see an instance above under 3). From this point of view, an attempt was made by the one of us and P. N. Leech to rearrange β-triphenylmethyl-β-methyl hydroxylamine $(C_6H_5)_3C.N(CH_3)OH$ by the same methods that had proved successful with triphenylmethylhydroxylamine $(C_6H_5)_3C.NHOH$. Indications of a rearrangement and the formation of a methylaniline derivative were indeed obtained,[12] but the evidence was quite uncertain, being limited to a general color reaction for monoalkylanilines and to some melting-point determinations of an acetyl derivative of the aniline.

We have now succeeded in establishing the fact that β-triphenyl-methyl-β-methyl hydroxylamine is rearranged in a perfectly normal fashion by the action of phosphorus pentachloride on the ether solution of the hydroxylamine. The compound formed in the reaction was shown to be $(C_6H_5)_2C(OH)N(CH_3)(C_6H_5)$ [or its chloride $(C_6H_5)_2CCl.N(CH_3)(C_6H_5)$] by the identification of its products of hydrolysis, methylaniline and benzophenone:

$$(C_6H_5)_2CCl.N(CH_3)(C_6H_5)+HOH\rightarrow(C_6H_5)_2CO+C_6H_5N(CH_3)H_2Cl. \quad (7)$$

The formation of the more important component methylaniline was proved (1) by the analysis of its chloroplatinate (found, 31.40% platinum; theory 31.27%); (2) by the comparison of its hydrochloride with the hydrochloride of a known preparation of methylaniline (its melting-point was 119°–122°, that of the pure hydrochloride the same, and the melting-point of a mixture of the two salts the same); (3) by the navy blue color produced by the action of bleaching powder solution on the methylaniline. To these tests may be added the melting-point observations made with Leech on the acetyl derivative of the base. The benzophenone formed was also isolated and identified by its melting-point (47°).

It was further shown that chlorophenyl-diphenylmethyl-β-methyl hydroxylamine $(ClC_6H_4)(C_6H_5)_2CN(CH_3)OH$, like the corresponding chlorophenyl-diphenylmethylhydroxylamine $(ClC_6H_4)(C_6H_5)_2C.NHOH$, shows a migration of the phenyl group in a fraction of the rearranging molecules, a migration of the chlorophenyl radicle in the remainder. The molar ratio of methylaniline to chloromethylaniline was 75/25.

It is, therefore, now for the first time an established fact that sub-

stances which could not form simple univalent nitrogen derivatives may be rearranged in the same fashion as those which would easily lead to such derivatives and the theory of the rearrangements must now count with this fact. Whether we actually have a direct exchange of radicles

$$(C_6H_5)_2C(C_6H_5)—NCH_3 \rightarrow (C_6H_5)C(OH)—NCH_3(C_6H_5) \qquad (8).$$

as postulated by Beckmann[20] for the analogous rearrangement of oximes $(C_6H_5)_2C{:}NOH$, or whether in both of these cases the rearrangements go by way of *salts* of univalent nitrogen derivatives, will be considered in detail in another place. It must suffice to point out here that whichever path the rearrangement follows, the final result is that the positive hydroxyl group of the hydroxylamines and of the oximes undoubtedly becomes a negative group in the course of the reaction: it still would be the unstable radicle in the original molecule and the satisfaction of its tendency to become negative would still be the prevailing influence in the rearrangement (see part I).

THE REARRANGEMENT OF TRIPHENYLMETHYL AZIDES AND OF SYM. DI-TRIPHENYLMETHYLHYDRAZINE

WITH JAMES KUHN SENIOR

While there may be some question as to the mechanism of the rearrangement of hydroxylamine derivatives (see above), the theory of a direct exchange of radicles (equation 8) would be quite inapplicable to the rearrangements of azides. Triphenylmethyl halogen amines and hydroxylamines having been found, in this laboratory, to undergo normal rearrangements, the problem of the rearrangement of triphenylmethylazides $(C_6H_5)_3C.N(N_2)$ was undertaken by us. It was intended to settle three main questions. The first two were: (1) Whether a rearrangement could be effected at all; (2) Whether heat alone would not be sufficient to effect the rearrangement, this being the agent indicated by the univalent nitrogen theory of the rearrangement:

$$(C_6H_5)_3C.N(N_2) - N_2 \rightarrow (C_6H_5)_3C.N \rightarrow (C_6H_5)_2C{:}NC_6H_5. \qquad (9)$$

Both of these questions were easily answered in the affirmative. When the azide is heated to a high temperature, it is decomposed, with a loss of nitrogen. The course of the rearrangement was followed by the hydrolysis of the product and identification of the substances thus formed, namely aniline and benzophenone. These are the compounds which phenylimido benzophenone $(C_6H_5)_2C{:}NC_6H_5$ (equation 9) should

form (equation 4). The aniline obtained gave the characteristic bleaching-powder test and formed a chloroplatinate which was analyzed (33.35, 32.32% platinum found, theory 32.78%); the benzophenone was identified by its melting-point 47.5°, which was unchanged when the product was mixed with some known, pure benzophenone.

(3) The third question raised in this investigation concerned itself with the following relations: in the rearrangement of a substitution product of triphenylmethyl azide, such as $(ClC_6H_4)(C_6H_5)_2C.N(N_2)$, a certain part of the substance would form an aniline derivative $(ClC_6H_4)(C_6H_5)C{:}NC_6H_5$, another part would yield a derivative of chloroaniline $(C_6H_5)_2C{:}NC_6H_4Cl$ (see above). The ratio may be easily ascertained by hydrolysis of the product and titration of the anilines with bromine. Now, the same situation exists in regard to the rearrangement of the corresponding hydroxylamine $(ClC_6H_4)(C_6H_5)_2C.NHOH$ and the chloroamine $(ClC_6H_4)(C_6H_5)_2C.NHCl$. If in each instance the path to the rearrangement lies through an intermediate univalent nitrogen derivative, we should have the identical product $(ClC_6H_4)(C_6H_5)_2C.N$ in each of these cases, the azide losing nitrogen, the hydroxylamine water and the chloroamine halogen acid. The environment of the univalent-nitrogen derivative would be different in the three cases on account of differences in the physical state, the temperature, etc. But since the rearrangement is an intramolecular one, it might be little affected by environment, once the compound $(ClC_6H_4)(C_6H_5)_2C.N$ is formed. A comparison of the ratios of anline to chloroaniline formed in the different series could then shed some light on the question of the probability of the formation of the hypothetical intermediate univalent nitrogen derivative. On account of the difficulty of securing experimental material only a beginning has been made on this phase of the problem of these rearrangements. We have already found, however, that in the rearrangement of $(ClC_6H_4)(C_6H_5)_2C.N(N_2)$ the ratio of aniline to chloroaniline formed is 70/30 in molar terms. This compares with the ratio[12] 71/29 found for the rearrangement of $(ClC_6H_4)(C_6H_5)_2C.NHOH$. The result speaks for itself.

That hydrazine and hydroxylamine are fundamentally similar compounds[21] is a well-established fact, shown both by their reactions and also especially by their forming parallel derivatives, of similar properties, with a great many organic substances. One strange difference in the chemistry of these derivatives, as thus far developed, seems to have been overlooked, and that is that molecular rearrangements of hydrazines, corresponding to the rearrangements of hydroxamic acids, ketoximes, triphenylmethylhydroxylamines, etc., have not been observed, or,

apparently, even attempted; for the 'benzidine' and related rearrangements of hydrazines are of a different type. This seemed the more strange since derivatives of the third member[21] of the group, hydrogen peroxide, have been found to undergo the same typical rearrangement[22] (see part I, equation 5) as the hydroxylamine derivatives.

When we undertook to bridge this gap in the chemistry of the hydrazine derivatives, no difficulty was anticipated in securing such rearrangements. Not only the fundamental analogy between the groups of compounds indicated this, but, from the point of view of the univalent nitrogen theory of the rearrangements, it seemed as if there should be no difficulty in removing ammonia from triphenylmethylhydrazine and thus starting a rearrangement:

$$(C_6H_5)_3C.NH.NH_2 - NH_3 \rightarrow (C_6H_5)_3C.N \rightarrow (C_6H_5)_2C{:}NC_6H_5. \quad (10)$$

We were very much surprised to find that all our attempts to cause a rearrangement of this hydrazine and of a great many others by means of acids or zinc chloride (reagents to combine with ammonia) were complete failures. Although the experiments are being continued in spite of the original failures, these were sufficiently numerous and impressive to lead us to reconsider in greater detail the theoretical relations involved. Now, for rearranging chloroamines (*e.g.*, $(C_6H_5)_3C.HN^{-}\ {}^{+}Cl$), hydroxylamines (*e.g.*, $(C_6H_5)_3C.HN^{-}\ {}^{+}OH$) or peroxides (*e.g.*, $(CH_3)_2C\langle {}^{O-}_{O^{+}} \rangle$), the distribution of the charges on the atoms in the unstable parts of the molecules can be determined with a great degree of reliability.[12] For a hydrazide, this is not the case, no criteria having as yet been established by means of which we can distinguish, for instance, between the electromeric structures:

$$(C_6H_5)_3C.HN^{-}.{}^{+}NH_2 \quad \text{and} \quad (C_6H_5)_3C.HN^{+}.{}^{-}NH_2$$

Obviously, however, the question of the molecular rearrangement of triphenylmethyl hydrazine could very well, and probably would, depend altogether on its electronic structure. For instance, a decomposition $(C_6H_5)_3C.HN.NH_2 \rightarrow (C_6H_5)_3C.NH_2 + (HN)$ would not lead to a rearrangement and the electronic structure of the hydrazide might insure this decomposition in place of the one expressed in equation 10.

These considerations made it appear desirable to attempt the rearrangement of a symmetrical molecule, di-triphenylmethylhydrazine, about whose electronic structure there could be no doubt: $(C_6H_5)_3C.HN^{+}.{}^{-}NH.C(C_6H_5)_3$. A loss of triphenylmethylamine from either side would leave a univalent nitrogen derivative $(C_6H_5)_3C.N$ and rear-

rangement should inevitably follow. Whether this argument should ultimately be found to be altogether right or not, it is a noetworthy fact that it led us to the first realization of the rearrangement of a hydrazine derivative along these lines. From the product of the action of zinc chloride on the symmetrical hydrazine, aniline was obtained and identified (by a positive chloride of lime test; by analysis of its chloroplatinate, which gave 32.80% platinum, theory 32.78%). The question as to the mechanism of the rearrangement awaits the identification of the further products of the reaction, but aniline could be formed only by a migration of a phenyl group from carbon to nitrogen, the type of rearrangement we desired to produce. This is, we believe, the first rearrangement of a hydrazine along the lines so characteristic for the hydroxylamine derivatives, which have been known for forty years. It is significant that the way to its accomplishment was pointed by the application of Thomson's original theory of electron valences to the subject of these rearrangements.

[1] J. J. Thomson, *Electricity and Matter*, 1905, and *Corpuscular Theory of Matter*, 1907.

[2] L. W. Jones, *Amer. Chem. J.*, 50, 440 (1913); *Ibid.*, 48, 1 (1912).

[3] J. Stieglitz and P. N. Leech, *J. Amer. Chem. Soc.*, 36, 272 (1914) and J. Stieglitz, *Ber. d. Chem. Ges.*, 43, 782 (1910); 46, 2149 (1913); *Amer. Chem. J.*, 46, 327 (1911).

[4] See the bibliography of the theory of the rearrangements, Stieglitz and Leech, *loc. cit.*

[5] See a preliminary report, J. Stieglitz and Isabelle Vosburgh, *Ber. D. chem. Ges.*, 46, 2151 (1913); also the following article.

[6] Th. Seliwanow, *Ibid.*, 25, 3617 (1892); F. Lengfeld and J. Stieglitz, *Amer. Chem. J.*, 15, 215, 504 (1893).

[7] W. A. Noyes, *J. Amer. Chem Soc.*, 23, 450 (1901); J. Stieglitz, *Ibid.*, 23, 796 (1901); Walden, *Zs. physik. Chem.*, 43, 385 (1903).

[8] J. J. Thomson, *Phil. Mag.*, 27, 757 (1914).

[9] A. A. Noyes, *Carnegie Inst. Publications*, 63, 351 (1907).

[10] W. C. Bray and G. E. K. Branch, *J. Amer. Chem. Soc.*, 35, 1440 (1913).

[11] G. N. Lewis, *Ibid.*, 35, 1448 (1913).

[12] J. Stieglitz and Leech, *loc. cit.*

[13] J. Stieglitz, *J. Amer. Chem. Soc.*, 30, 1797 (1908). See also W. A. Noyes, *loc. cit.*

[14] For the theory of the intermediate formation of univalent nitrogen derivatives, see J. Stieglitz, *J. Amer. Chem. Soc.*, 18, 751 (1896); 29, 49 (1903), and the literature references, Stieglitz and Leech, *loc. cit.*

[15] A. Werner, *Ber. D. chem. Ges.*, 25, 33 (1892); 26, 1562 (1893); 29, 1153 (1896); M. Kubara, *Mem. Coll. Sci. Engin.*, Kyoto, 6, 1 (1913).

[16] J. Stieglitz and G. Curme, *Ber. D. chem. Ges.*, 46, 911 (1913).

[17] J. Thiele, *Ibid.*, 44, 2522 (1911).

[18] J. J. Thomson, *Phil. Mag.*, 27, 784 (1914).

[19] J. Stieglitz and P. P. Peterson, *Ber. D. chem. Ges.*, 43, 782 (1910), and P. P. Peterson, *Amer. Chem. J.*, 46, 325 (1911).

[20] E. Beckmann, *Ber. D. chem. Ges.*, 19, 988 (1886); 27, 300 (1894).

[21] A. Angeli, *Chem. Zentralbl.*, 1910, II, 861.

[22] A. v. Baeyer, *Ber. D. chem. Ges.*, 32, 3627 (1899).

ECOLOGY OF THE MURRAY ISLAND CORAL REEF

By Alfred Goldsborough Mayer

DEPARTMENT OF MARINE BIOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, February 13, 1915

A quantitative ecological study was made of the Madreporian corals of the fringing reef of Maër Island, the largest of the Murray Island Group, which lies in Torres Straits, Australia, in 9°55′ S. Lat.; 144°2′ Long. E. from Greenwich.

Maër Island is volcanic and has broken through an old limestone-bearing platform which is now submerged to a depth of 15–30 fathoms, and upon which the modern coral reefs of Torres Straits have grown.

The volcanic center of the Island is surrounded by a modern fringing reef which has grown seaward over its outer edges from the shores, and is now 1800–2200 feet wide on the windward (S. E.) side of the Island, and only 175–780 feet wide on the leeward (N. W.) side. It is wide in regions exposed to the full force of the breakers, and where it is not interfered with by silt from the three principal streams of the Island; while on the other hand in regions protected from breakers or inundated by sand or silt the fringing reef is narrow. Hurricanes are unknown and thus the corals grow on uninterrupted by periods of wholesale destruction such as affect the reefs of the West Indies, and most parts of the Pacific region.

A line running S. 39° E., and 1869 feet long, was surveyed across the S. E. reef flat of Maër Island. The shore end of this line was 1496 feet in a N. E. direction from the mouth of Haddon brook. Squares 50 feet on the side (2500 feet in area) were then surveyed and staked out at intervals of about 200 feet along this line; and all living coral heads growing within these staked areas were counted.

This reef flat is peculiar in that the water is dammed by the lithothamnion ridge which extends in a narrow barrier along the seaward, breaker-washed edge of the flat. Thus at low tide the water over the reef flat becomes a marine basin about two miles long, 1680 feet wide and only about 18 inches deep; but the water being impounded by the lithothamnion ridge, the reef flat is never laid bare even by the lowest spring tides. About 3,600,000 living coral heads are found upon this submerged area. For the first 370 feet out from shore there are no corals. The following table gives the number of living coral heads found on each 50-foot square (2500 square feet) at intervals of about 200 feet apart across the S. E. reef flat or Maër Island.

Distance[1]	Living[2]	Species[3]
400	3	2
525	110	11
625	126	9
825	413	13
1025	529	15
1225	962	9
1425	1838	18
1650	1512	27
1750	201	15

[1] Distance in feet of the center of the square from shore.
[2] Number of living coral heads growing within each square (2500 square feet).
[3] Number of different species of corals on each square.

About 40 different species of corals are found growing within these squares. Of the 22 genera of corals found on these squares, four constitute 91% of the living coral heads. Thus: Porites 38%, Seriatopora 25%, Acropora 18%, and Pocillopora 10%.

Seriatopora is the most successful coral of the calm waters of the middle zone of the reef flat 1100 feet from shore where it covers 40% of the area of the bottom, and constitutes 70% of the entire number of living coral heads. In this region where Seriatopora is dominant all other species of corals are reduced in number. Yet Seriatopora cannot live within 500 feet of the shore, owing to the high temperature of the water in this region, nor can it survive in places more than 1650 feet from shore due to the destructive action of the breakers upon its fragile stems.

The coral heads are most densely clustered in a region about 200 feet inward from the usual inner 'wash' of the breakers, but an even greater variety of *species* of corals are found to the seaward of this place where the water is strongly agitated by the surges. Thus in the place where coral growth is most dense there are only 18 species, but 200 feet to seaward of this zone there are 27 species, although owing to the rough water the coral stocks are much broken and are either large massive heads capable of resisting the waves or small ones protected within crevices.

Of the 22 genera from the squares over this reef flat 13, or more than half, are confined to the seaward parts of the flat and do not commonly appear within 1200 feet of the shore. In fact only two species, Bernard's 'Porites No. 12' and a form allied to 'Siderastrea' sphaeroidalis Ortmann are practically confined to the shore flats within 1100 feet of the beach.

Temperature is the dominant factor, and is even more important than silt in determining the habitat of corals. The most sensitive

genera such as Acropora, Seriatopora, and Pocillopora are killed at from 36.2°–37°C., while Siderastrea and some species of Porites can withstand heat up to 38° or even 38.5°C. Generally speaking those forms which are sensitive to high temperature are correspondingly affected by being smothered under mud, or subjected to the influence of CO_2. This suggests that high temperature produces death by asphyxiation as postulated by Winterstein. There are however some exceptions, for certain corals such as Favia fragum and Mæandra æreolata die at a relatively low temperature but are quite resistant to asphyxiation. It appears from experiments conducted at Tortugas, Florida, that some corals can adjust themselves to a wide range of metabolic activity and can thus survive in a reduced oxygen supply by corresponding lowering their metabolic processes. Others, however, cannot effect this adjustment and must live at a fairly uniform rate of metabolism.

The more sensitive corals such as Acropora, Seriatopora, Euphyllia, or Pocillopora, which are all off-shore, pure water forms, are killed by being buried eleven hours under the mud, whereas the shore flat species of Porites, Siderastrea, and Mæandra can survive being buried from 24 to 73 hours.

The corals of this Australian reef which are never subjected to cold can nevertheless withstand low temperature quite as readily as can the the corals of the 'cold devastated' reefs of Florida; and conversely the Florida corals can withstand high temperatures quite as well as do those of Australia. In other words corals are, physiologically speaking, of similar constitution whether in the Atlantic or the Pacific; and natural selection has apparently not operated to improve their cold-withstanding or heat-resisting powers. The reef building forms must live in water which is warmer than 15°, and cooler than 38°C.

It was found that the shallow waters of the Maër Island reef flat receive most of their heat by direct radiation from the sun and lose it by radiation into outer space at night. Thus at 3 p.m. the water is from 1.2° to 6.7° higher than the air, while at 6 a.m. it is from 0.3° to 3°C. lower than the air temperature.

The range in water temperature was thus greater than that of the air; for during September and October the extreme range in air temperature was only 3.4°C. whereas the water of the reef flat 200 feet from shore ranged through 12.5°, and at 1860 feet from shore, in the breakers, the range was only 3.5°C.

Thus the waters of the shore flats within 400 feet of the beach must become too hot for coral life during the calms of the 'north west season' between November and April.

There is clear evidence of the submarine solution of a thickness of about two feet of limestone in the middle region of the Maër Island reef flat between 400–1600 feet from shore; but this may be chiefly due to the carbon dioxide washed outward from the densely forested shores of the Island during the rainy season, and not to the sea water as such.

Indeed the experiments and observations of Dole, and of Vaughan would lead to the conclusion that the sea water of coral reef lagoons lacks free CO_2 and is therefore probably incapable of dissolving limestone.

All species of reef corals survive without apparent injury an immersion for 4 to 5 hours in sea water diluted with an equal volume of rain water, and many species can withstand 11 hours of this treatment, and thus it appears that even torrential rains cannot be an important factor in the destruction of the reef flat corals of Maër Island through dilution of the water, for the tidal range is about seven feet and the incoming sea water would soon offset any dilution due to rains.

The injurious effects of rains upon coral reefs is due solely to the silt which they cause to be washed outward over the flats.

The research of which the above is an abstract, will be published by the Carnegie Institution of Washington.

CHANGES IN SHADE, COLOR AND PATTERN IN FISHES AND THEIR BEARING ON CERTAIN PROBLEMS OF BEHAVIOR AND ADAPTATION

By S. O. Mast

ZOOLOGICAL LABORATORY, JOHNS HOPKINS UNIVERSITY

Presented to the Academy, February 3, 1915

It is well known that the surface of many organisms is in appearance much like their environment. That is, the organisms simulate their surroundings and consequently are more or less inconspicuous. In some of these organisms the characteristics which produce such simulation are fairly permanent and the creatures correspond in appearance with, what may be termed, the general average of their environment. In others these characteristics may change rather rapidly, in such a way that the animals appear almost continuously like their surroundings, no matter how much they may change. These phenomena have always excited a lively interest in naturalists and a great amount of work has been done on them. They have been particularly prominent in discussions bearing on evolution and consequently most of the investigations concerned questions of function and origin and development, both individual and racial.

This paper however deals primarily with the extent and the accuracy of the process of adjustment to the environment, the factors involved in it, with its bearing on certain associated problems in behavior. Nearly every investigator assumes that the adjustment to the environment in an organism serves to conceal it from other organisms. This assumption rests upon the further assumption that vision in animals is similar to that in man. Thus we have the problem of vision closely associated with the question concerning the function of simulation of the environment. There are also some other associated problems in behavior which are of interest, particularly the one concerning the influence of experience on the rate of adjustment.

1. Extent, accuracy and nature of simulation. Simulation of the environment in fishes involves changes in shade, changes in color and changes in pattern. Nearly all fishes assume a light shade over bright bottoms and a dark shade over dark bottoms, and a considerable number assume the predominating colors of the environment, but in only a few does the pattern change so as to harmonize with that of their surroundings.

In some of the flounders simulation of the background is probably more extensive, accurate, and rapid than in any other animals. Two of these forms, Paralichthys and Ancylopsetta, were thoroughly studied regarding this response. It was found, in brief, that in glass dishes, either on artificial or on natural backgrounds, changes occurred in the skin such as to make the animals very closely resemble the background in shade, color and pattern regardless of great variations in these respects. On a white background they became remarkably nearly white; on gray they became gray of practically the same shade; and on black they became very nearly black. On colored backgrounds varying from dark blue to dark red they assumed colors very similar to those of the background in all but the red. On backgrounds having small figures the figures in the skin became correspondingly small and on those having large figures they became, within certain limits, correspondingly large. There is, however, no indication of an actual reproduction of patterns. While the size of the light and the dark areas in the background and the relative amount of surface covered by them have a profound effect on the pattern produced in the skin, their form and special interrelationship have, within wide limits, no effect.

2. Rate of adaptation to the background. The time required to produce adaptive changes in the skin in Paralichthys and probably also in other genera varies greatly. Under some conditions changes resulting in maximum adjustment have been observed to occur in two minutes

or less, under others it requires several days. In general it is considerably longer for old specimens than for young. It is much longer in individuals kept continuously on a given background than it is in those frequently changed from one kind to another. This is clearly shown by the following: An individual, after having been in a white granite pan continuously for two weeks and long since maximum white, was transferred to a black pan, August 18, 2.05 p.m. At 4.30 p.m. it was about one-half maximum black; August 19, 12 m. about three-fourths maximum black; August 22, 10 a.m. nearly maximum black; August 23, 10 a.m. maximum black. This same individual, after having been frequently transferred from white to black and vice versa, from August 23 to August 30, was taken from the white background where it was maximum white and put into the black pan at 7.27 a.m. One minute later, 7.28, it was already five-sixths maximum black, and after one minute more, 7.29, it was maximum black. The change from black to white however was never observed to be so rapid as this; in all of the experiments it required an hour or more. Thus while it required five days to produce a complete change in the skin from white to black after continuous sojourn of two weeks on white, it required only two minutes after repeated transfers from one to the other.

Changes in color require, in general, much more time than changes in shade or changes in pattern. There is however much variation regarding this among the different colors. Yellow for example, is a color that the fish assume much more readily and rapidly than green or blue. This may be due to the fact that yellow ordinarily predominates in their environment.

3. Factors involved in adjustment to the background. a. Chromatophores: In the skin of the flounders there are colored cells known as chromatophores. Some of these are black and others are yellow of various shades. Associated with these there are other cells which contain numerous highly refractive crystals said to be guanin. These cells are called iridocytes. They appear pure white in reflected light. The pigment in all or in any group of the chromatophores can be concentrated in small globular masses, or it can be spread out so as to cover relatively large surfaces. Moreover, the iridocytes may assume such a position as to hide all or any portion of the pigment in the chromatophores. Thus, changes in shade, color and pattern are produced by reactions in these bodies. And these reactions are regulated by stimuli received through the eyes, as the following experimental observations demonstrate.

b. Eyes: Specimens with the anterior end on white and the posterior

end on black become uniformly maximum white over the entire pigmented surface, with the anterior end on black and the posterior end on white or on a glass plate with intense light reflected through it from below they become uniformly maximum black; with one eye on white and the other on black they assume an intermediate shade (uniform gray) over the entire surface. Specimens, with one eye on a background having a given pattern and the other eye on a background having a different one, assume uniformly over the entire pigmented surface of the body, a pattern intermediate between those assumed when entirely on either background. Careful observation shows that this pattern really consists of a sort of superimposition of a coarser and a finer pattern; for large areas similar to those ordinarily produced when the fish is entirely on the coarser grained of the two backgrounds can still be faintly seen, as well as small areas similar to those ordinarily produced when the fish is entirely on the finer grained background. Thus the configuration in the skin when one eye is on a background having a given pattern and the other on one having a different pattern consists of a superimposition of the configuration produced by each of the two different backgrounds acting alone. The influence of the light received by each of the two eyes is evidently distributed over the entire body and the resulting pattern in the skin is due to a combination of the specific effect of stimuli received from the two eyes.

On a background consisting of any combination of black and white, only black and white is seen in the skin, no color whatever. But if a yellow card is placed within 3 cm. of the anterior end of the fish it becomes strikingly yellow. All of these results indicate that if light has any direct effect on the chromatophores it is insignificant in the process of adjustment to the background. This conclusion is further supported by results obtained with blind specimens.

The removal of either eye interferes but little with the activities of flounders. If the operation is carefully performed the wound heals in a few days and the animals respond normally. They move about and feed apparently with the same degree of freedom as they do with both eyes functional and there is no difference in the extent or the rate of adaptation to the background, neither in color nor in shade nor in pattern.

Specimens with both eyes removed also learn to get around in the aquarium without serious difficulty; but they do not simulate the background, either in shade, color, or pattern.

c. Direction of the light: Specimens on a white background, if they respond at all, become maximum white even if the intensity of the light is so low that they can scarcely be seen and the background appears

dark gray to the human eye. On a gray background, however, even if it appears much lighter than the white in low light intensity, they become gray, on a black, black, etc. If however conditions are so arranged that the eyes receive no light direct from above, that is, receive only light reflected from the background, they become maximum white no matter what the shade or pattern of the background may be. This shows that the changes in shade and pattern and probably also color are in some way dependent upon the relation between the light received directly and that received by reflection from the background.

In accord with this, if the light from above is abnormally strong the fish ought to become dark on a white bottom. Apparatus was devised by means of which such illumination could be produced but it was found that when the light from above was abnormally strong the fish did not respond at all. The significance of this negative result is still somewhat problematic.

d. Comparison of the skin with the bottom: In specimens with the ventral eye removed it was possible to keep the anterior part, with the exception of the eye, so thoroughly covered with sand that the fish could not see any of the skin. Under these conditions adaptation occurred just as accurately and rapidly as it did when the skin could be seen. This shows that the skin is not necessarily compared with the bottom by the fish in the process of adaptation.

4. Selection of background. After flounders have been, for some time, on a given background they tend to return to this background when put onto another near it. That is, they tend to select the background on which they are least conspicuous. This was conclusively proved with reference to shade and fairly definitely with reference to color. The response is, however, of such a nature that, under normal conditions, it is probably of little value in concealing the animals; but it does throw some light on the question of vision referred to later.

5. Biological significance of simulation of the background. Some hold that the phenomenon of simulation is purely accidental and that it has no biological value: others maintain that it functions as a protection from enemies or in capturing prey; a few contend that it serves chiefly to regulate the temperature of the body; and some even hold that it functions in all of these ways and in still others. Unfortunately, however, none of these ideas are supported by experimental evidence. In no case has it ever actually been demonstrated that the response in question has any value whatsoever. This statement includes the results obtained in work on the flounders, although the experiments outlined have as yet been only partially completed.

6. Vision. Since the changes in the skin which result in simulation of the background are controlled by stimuli received through the eyes, the nature and accuracy of the process constitutes an excellent criterion of vision in so far as this term may be used in a purely objective sense.

On the basis of this criterion it was found that, in regard to shade and color, vision in fishes is essentially the same as it is in man; but that in regard to size it is less acute. It was found that flounders distinguish between dots 2 mm. and 3 mm. in diameter respectively, that they recognize dots 1 mm. in diameter but that they do not recognize those 0.5 mm. in diameter.

The most interesting and convincing evidence that we have, however, regarding vision, refers to motion. By means of a background consisting of a rotating disk composed of alternate black and white sectors, it was found that the fusion-rate of images in flounders corresponds very closely with that in human beings. This seems to show conclusively that, in regard to motion, vision in fishes is as acute as it is in man.

There has been so much contention concerning color-vision in animals that it may be well to add a few words in explanation of our evidence bearing on it.

On a background containing only gray or black and white no color is produced in flounders regardless of the shade or pattern or the intensity of the light. Simulation in color is consequently dependent upon the length of the waves of light, not upon difference in its intensity. This is one of the essential characteristics of color-vision in man. We have, however, still further evidence that bears on this matter. Flounders adapted to a given color tend to select a background of the same color: and this selection is of such a nature that it cannot be accounted for on the basis of difference in the intensity of the light reflected by the different colors. It must, therefore, be associated with the length of the waves. Thus the contention that fishes have color-vision is supported both by the reactions of the animals and by the reactions of the chromatophores in the skin. This evidence has, of course, no direct bearing whatever on the problem of color-sensation.

This work is a contribution from the U. S. F. Biol. Sta., Beaufort, N. C., U. S. A. Presented in this form by permission of the Commissioner of Fisheries it is to be published in full, with numerous photographs and autochromes, in the *Bulletin of the U. S. Bureau of Fisheries.*

SOME EXPERIMENTS ON SPERMATOGENESIS IN VITRO

By Richard Goldschmidt
(of the Kaiser Wilhelm Institut für Biologie, Berlin)
OSBORN ZOOLOGICAL LABORATORY, YALE UNIVERSITY
Presented to the Academy, March 3, 1915

Unforeseen circumstances that forced me to interrupt my normal work offered a chance for trying some experiments, that I have had in mind for some time, to apply the method of tissue culture to the study of sex-cells with the purpose of rendering the history of these cells accessible to an experimental analysis. The results obtained in this direction during the past winter constitute of course nothing more than the first steps into a very promising field of research. For the experiments the sperm-cells of the moth, Samia cecropia L., were used. The method of cultivation is simply Harrison's method. By tearing the testis of the pupa the thousands of sperm follicles (spermatocysts) are isolated and can easily be mounted in a hanging drop of haemolymph. Naturally the whole procedure must be carried out strictly aseptically. In November most of the follicles contain spermatogonia or young spermatocytes during the growth-period. They could be kept alive in the cultures about three weeks, and during this time a great many follicles finished the process of spermatogenesis up to the full grown spermatozoa under the eye of the observer. The time required depends, as was to be expected, upon the temperature. It seems that the stages from the spermatogonia up to the maturation divisions need weeks for their development but maturation and spermatogenesis occur in a few days even in one day when kept warm. The most striking, although in no way surprising result of the observations is the complete harmony between the facts as observed in the living cell and those described from preserved specimens. The formation of the axial filaments in the young spermatocytes and their subsequent behavior, as well as the mitochondria and their distribution during the maturation divisions, could readily be followed. The interesting and hitherto unobserved behavior of the axial filaments in the formation of the regular sperm-bundles could also be made out. In addition interesting observations were made upon the behavior of the follicle membrane after the death of the sperm cells. At this time an extensive outgrowth of these cells begins, forming a kind of tissue which grows luxuriantly for some weeks more.

The observations made upon normal spermatogenesis suggested the hypothesis that the specific changes undergone by a sperm cell in its development into that strange thread-like spermatozoon may be due

not so much to internal changes inside the cell as to changes in the physical constitution of the surrounding medium, controlled by the follicle membrane, forcing the cell to react in that specific way. In order to test this hypothesis experiments have been made in influencing the osmotic conditions inside the follicular membrane. One of the clearest results was that hypertonic condition in the haemolymph forces all sex-cells from spermatogonia to spermatids to grow out in one direction, namely, against the follicle wall. The resistance of the latter forces the cell to grow along its inner surface so that finally the follicle appears as a whorl of thread-like cells. The amount of growing out depends upon two factors: the age of the cell and the hypertonicity of the medium. Spermatocytes during or before the maturation divisions form long spermatozoa-like threads but with inverse polarity, the axial filament on the inner surface of the cell (present as it is well known in butterflies from the spermatocyte stage on) not being involved at all. The whole process proved to be reversible without change of the medium, as were also the slight changes produced by a hypotonic medium, since after some days all cells returned to their normal form. This means that the follicle membrane has the faculty of active control of the osmotic conditions inside the follicle. In addition to these experiments an effort was made to induce the cells of different age to grow in the normal direction along the axial filaments. Up to the present only the first steps have been obtained by treatment with diluted methyl alcohol. Another normal process that could be induced at a different point in the cell history was the formation of the axial filament. In these experiments interesting observations have been made upon the formation of flagella by the plasm of all sperm cells from the spermatogonia to ripe spermatozoa. They arise in Ringer's Fluid in a similar way as described for other cells by Merk, Kite, Oliver, Chambers, and they can be produced and made to disappear at will by change of temperature.

All these experiments together with the observations of the normal process point to the probability that the general processes of spermatogenesis are necessary reactions of the cells to a systematic regulation of the osmotic conditions on the part of the follicular membrane. The individual specific processes are caused by the specific properties of the reacting cells. This affords us the possibility of an experimental attack upon other closely related problems, viz., the function of Sertoli's cells, which may prove to be the same as the function of the follicle cells in the present case, and the problem of apyrene spermatozoa, which, according to some of my observations, may prove nothing else than a *lusus naturae*, an abnormality produced by slight changes in the physi-

cal properties of the follicle membrane that might readily occur under ordinary conditions. These questions will be discussed in the full account of the work to be published in the *Archiv für Zellforschung*.

GROWTH AND VARIATION IN MAIZE

By Raymond Pearl and Frank M. Surface

BIOLOGICAL LABORATORY, MAINE AGRICULTURAL EXPERIMENT STATION

Presented to the Academy, February 23, 1915

The Problem. The investigation reported in this paper is an attempt to analyze the normal variation of an organism in a particular case, from the standpoint of *Entwicklungsmechanik*. It was suggested in the first instance by the study of variation and differentiation in *Ceratophyllum* made by Pearl some years ago. We have here attempted to approach the problem of *inter*-individual variation from the same point of view and with similar methods to those applied to the problem of *intra*-individual variation in the case of *Ceratophyllum*.

The problem and the point of view may be most clearly defined by considering briefly certain well-known, indeed obvious, facts about variation. If one brings together a homogeneous group of individual plants or animals and measures the same character in each individual, there may be formed from the resulting data a characteristic variation curve for that group and character. The precise form of this curve, as well as the location in it of any particular individual, are functions of two basic variables. Of these one is the hereditary or germinal constitution of the individual. The other is the complex of environmental stresses and strains, which, each acting on the individual during its ontogeny have influenced the end result of the activity of the hereditary determiners or genes.

Now it is altogether usual in discussions of variation and heredity to take the two end terms of the ontogenetic series, the gene on the one hand and the adult soma on the other hand, as things given. What comes between the two is neglected. But clearly what goes between is a part of the very essence of heredity itself.

In any group of adult individuals each one will show some particular variation, in the sense of a deviation from the typical condition of the group. We take it to be one of the final objects of investigations in genetics to find out why (in the sense of locating the essential causal factors involved) a particular individual A exhibits a particular variation a, and not some other variation out of the indefinitely large number

of possibilities. By means of Mendelian modes of research considerable progress has been made towards answering this question for many kinds of qualitative variations. For fluctuating or quantitative variations also, progress has been made through the researches especially of East and his associates, following those of Nilsson-Ehle on colors determined by multiple factors.

What we have tried to do in the present investigation is, by studying the growth of the individual, to analyze the adult variation curve into its component elements. We have attempted, in other words, to make a beginning at an understanding of the developmental physiology of the genes concerned in the production of the characters studied.

A given individual, at an early stage of growth, may exhibit an exceptional condition of a character, as compared with other individuals of the same age or stage of growth. It may be, for example, very short, the shortest plant in the whole population at three weeks of age. Will this same individual be the shortest plant in the adult population, after all growth is completed? If not, where is it in the adult variation curve, and how did it get there? This example will give a concrete idea of the general mode of approach to the problem of variation followed by this paper.

Results. Measurements were made at twice-a-week intervals of the height of each individual of three series of plants of a variety of sweet corn used in other experimental work in this laboratory and described in earlier papers. The heights were measured to the tip of the tallest leaf. In addition to these, separate sets of measurements were also made of the tassel height as soon as the tassels appeared. The growth curves obtained by plotting the mean height at each measurement are relatively smooth. After July 3, the time of tasseling, the plants grow in height much faster than before. Growth in height ceases entirely as soon as the tassel blooms.

The relative variability considered for the whole season shows a marked progressive diminution. It thus follows the general growth law of diminishing variability. Considered in detail, however, the relative variability first shows an increase. After June 19 there is a rapid decrease until the time of tasseling. During the period of tasseling there is a very rapid increase in the relative variability. This is followed by an equally rapid decrease. A stable condition is finally reached which is some 10 per cent lower than the variability at the beginning of the season.

The quantitative relations of tasseling to the growth and variability of the height of corn plants are very similar to the quantitative relations

of puberty to the growth of children. This does not necessarily mean that the two processes are analogous physiologically, but it is a fact of interest and worth further consideration from a physiological point of view.

It appears probable that the corn plant grows in a series of cycles. Each cycle is characterized by the special development of one set of organs. They are, in order, the root cycle, the leaf cycle, the tassel cycle, and the ear cycle.

With these general facts regarding the growth of the maize plant in hand the analysis of the variation in terms of individuals was undertaken. Individual plants and groups of plants having the same relative size at one stage of their growth, were followed as individuals through the re-

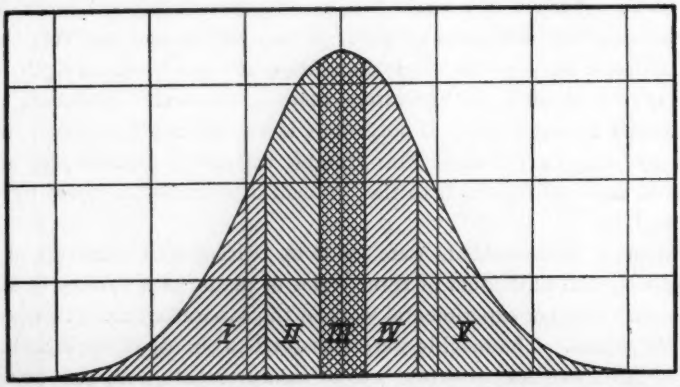


FIG. 1. DIAGRAM TO ILLUSTRATE THE DIVISION OF A FREQUENCY CURVE INTO QUINTILES. IN THIS DIAGRAM A NORMAL CURVE HAS BEEN DIVIDED INTO 5 EQUAL AREAS BY THE FOUR PERPENDICULAR SOLID LINES. THE FREQUENCY IN ANY ONE OF THESE AREAS IS EQUAL TO THAT IN ANY OTHER.

maining growth stages. We have endeavored to ascertain how such plants are distributed as to relative size in the successive growth stages and to discover the factors determining these distributions.

To study these questions it is necessary to have a measure of the *relative* size of the plants at each growth stage. For this, each distribution was divided into five equal areas or quintiles. In any distribution the relatively small plants are in quintile I and the relatively large ones in quintile V. Figure 1 illustrates the exact meaning of a quintile.

The problem was first approached by studying the quintile distribution of all the measurements, throughout the season, of a group of plants starting in a given quintile. It is shown that there is a strong tendency for the plants to remain in or near the quintile in which they started. As a measure of this tendency use is made of the root-mean-square devi-

ation of these observed distributions from the most probable distriburion of such measurements on the theory of chance. The tables of these constants are too extended to be given here, but the figures show that in every case (except one) the deviation from the theoretical mean is very much greater than would occur on the basis of chance. The deviation of the very small and the very large plants (quintiles I and V) are much greater than any of the others. Thus there is a much more marked tendency for the extreme plants to remain in the extreme classes than for the medium sized plants to remain in a particular medium sized class. There is a similar, though less marked, tendency for the plants *ending* in a given quintile to have remained in or near that quintile throughout their growth.

The second step in the analysis was to study the *mean quintile position* of each group of plants in the successive growth stages. Here the main conclusion is entirely clear. Extreme variants at the beginning of the season tend strongly, on the whole, to remain extreme variants during the whole season. At the same time such extreme variants regress slightly towards the general population mean as growth continues. This second tendency is, however, by no means so strong as the first.

The third step in the analysis of the variation curves was a study of the average relative size (mean quintile position) of the individual plants and of the variability of individual plants with respect to relative size. Definite quantitative evidence is presented showing that:

1. The observed differences in the manner of growth of individual plants and of groups of plants cannot be explained as the effect of external, environmental factors.

2. These differences are rather to be looked upon as the effect of internal factors.

3. The distribution of the average relative size (mean quintile position) of individual plants is such as to suggest the random distribution of these factors among the plants. The same thing is brought out by the distribution of the relative measurements of plants starting or ending with a given relative size (quintile).

4. The simplest method of explaining these facts is to regard the differences in the manner of growth as due to independent Mendelian factors which are distributed at random in any population of open fertilized maize plants. These factors would occur in the proportions found in a stable Mendelian population mating at random.

5. By assuming the presence of two independent growth factors and weighting each with the proper value, it is possible to obtain a theoretical distribution agreeing very closely with the observed distribution.

6. The interpretation of the growth of these plants by Mendelian factors is further strongly supported by the distribution of the standard deviations of the plants with different relative sizes. Thus it has been shown that the extreme plants which would be more nearly homozygous and for this reason less variable are, as a matter of fact, some 50 per cent less variable than the plants in the middle class after all allowance has been made for the difference in the size of the means.

This is a preliminary abstract of a paper with the same title now in press in the *Zeitschrift für induktive Abstammungs- und Vererbungslehre.*

ON A NEW GROUP OF BACTERICIDAL SUBSTANCES OBTAINED FROM HEXAMETHYLENETETRAMINE

By Walter A. Jacobs and Michael Heidelberger

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, February 17, 1915

Ever since the introduction of hexamethylenetetramine into medicine, this drug has attracted wide attention owing to its far-reaching possibilities as an internal antiseptic. In addition to its rapid appearance in the urine it is found after ingestion in practically all the body secretions, including even the cerebrospinal fluid. It is fairly well established that the antiseptic action of the drug is due to the liberation from it of formaldehyde by virtue of the acidity of the medium in which it acts, but that its diffusion throughout the body is a physical property of the undecomposed molecule. In other words, hexamethylenetetramine is but a vehicle for formaldehyde, and if the two conditions are satisfied, firstly, its appearance in large enough amount in the locality desired, and secondly, its cleavage to formaldehyde, a local internal antiseptic action might be expected. Owing, among other things, to the alkalinity of most of the body fluids the limitations set by these prerequisites are obvious.

Numerous attempts have already been made to achieve by proper chemical variation an improvement in the action of the drug. Such attempts, however, which have been confined almost entirely to combinations of hexamethylenetetramine with acids and phenolic compounds, have offered few advantages over the original drug. It seemed to us that more promising results might be obtained by a more direct variation of the hexamethylenetetramine molecule itself and a brief outline of the facts observed in the course of this work is the subject of the following:

The molecule of hexamethylenetetramine functions as a tertiary base,

and very readily adds organic substances containing aliphatically bound halogen, forming quaternary salts, a few of which are already described in the literature. In the present work over two hundred such preparations were made, and, after chemical characterization, subjected to biological study.

When tested against various strains of bacteria *in vitro* it was found that with but few exceptions these compounds were bactericidal, the bactericidal power varying with the character of the alkyl or aryl group introduced into the hexamethylenetetramine molecule. These substances are, on the whole, easily soluble in water, but on standing or warming the solutions rapidly decompose, and in the case of the aromatic derivatives, develop a very finely divided emulsion of oil droplets or a colloidal suspension which precipitates more or less readily according to the concentration. On investigation, these decomposition products proved to be the methylene derivatives of the aryl or alkyl amines corresponding to the halides originally used. In the case of the hexamethylenetetramine addition products of the halogenacetylamines these precipitates were more closely studied. In several instances the methyleneglycyl derivatives were isolated in pure crystalline form, and upon testing for bactericidal power, the results were found to agree closely with those obtained with the parent quaternary salts. We are, therefore, led to believe that at least in this type of hexamethylenetetramine derivatives, the bactericidal properties of the parent substances are due to the methyleneglycyl derivatives produced on decomposition, or more particularly, to the methyleneamino group contained in these decomposition products. Since all types of hexamethylenetetramine quaternary salts (with the exception of a few of aliphatic nature) seem to undergo this form of decomposition, we are inclined to believe that in general the methyleneamino compounds are responsible for the bactericidal effect. With the confirmation of this we are at present engaged.

The form in which these methyleneamino compounds separate from the solutions of their parent compounds is a contributing factor in the action of these substances. For when once precipitated from suspension, they cannot readily be redissolved unless they contain a solubilizing group. The use of hexamethylenetetramine in this manner would seem, therefore, but a convenient artifice for obtaining these substances in the form of emulsions or colloidal solutions, conditions necessary for their bactericidal action. In the presence of serum or protein this precipitation is in the majority of cases either diminished or prevented entirely. In this connection it may be stated that with a large number of the compounds the bactericidal effect was found to be but slightly inhibited

by serum or protein, a property essential for their use as internal antiseptics. The degree of bactericidal effect obtained depends not only upon the chemical nature of the compound but also upon the organism employed for the test. Comparative experiments with different bacterial species and with different varieties of the same species have demonstrated in many instances remarkable specificity. One class of substances, for example, killed *B. typhosus* in a dilution of 1 to 200 in two hours, whereas, in the same time it killed streptococcus in dilutions of 1 to 50,000 or more. The toxicity of the substances varies naturally with the character of the group added to the hexamethylenetetramine molecule.

A further study of the bactericidal properties of methyleneamino compounds in general is now in progress. A detailed description of the chemical and bacteriological work will follow in the appropriate journals.

CHEMICAL AND PHYSIOLOGICAL STUDIES OF A MAN FASTING THIRTY-ONE DAYS

By Francis G. Benedict

NUTRITION LABORATORY, CARNEGIE INSTITUTION OF WASHINGTON

Presented to the Academy, March 9, 1915

The important relationship between numerous diseases and the various stages of nutrition and the fact that many pathological cases border on complete or nearly complete inanition make a study of the physiology and chemistry of normal fasting of special importance to the physiologist and the clinician alike. It is rarely practicable to make exhaustive and simultaneous observations in several physiological and chemical fields of study with a subject undergoing a prolonged fast but the arrival in Boston in 1912 of a Maltese, A. Levanzin, who wished particularly to fast thirty-one days under strict scientific observation, presented such an opportunity. It was thus possible to supplement earlier data secured with men during relatively short periods of fasting (2 to 7 days) in the chemical laboratory of Wesleyan University. (Benedict, Carnegie Institution of Washington Publication No. 77, 1907.)

In accordance with a previously arranged program and in coöperation not only with the entire scientific and computing staff of the Nutrition Laboratory but with several members of the faculty of the Harvard Medical School, an extensive study of this man was made. The experiment began April 14, 1912, with a preliminary period of three days with food, followed by thirty-one days of complete abstinence from

food and drink other than distilled water, and finally a three-day realimentation period. In all save the last period the experiment was most successful.

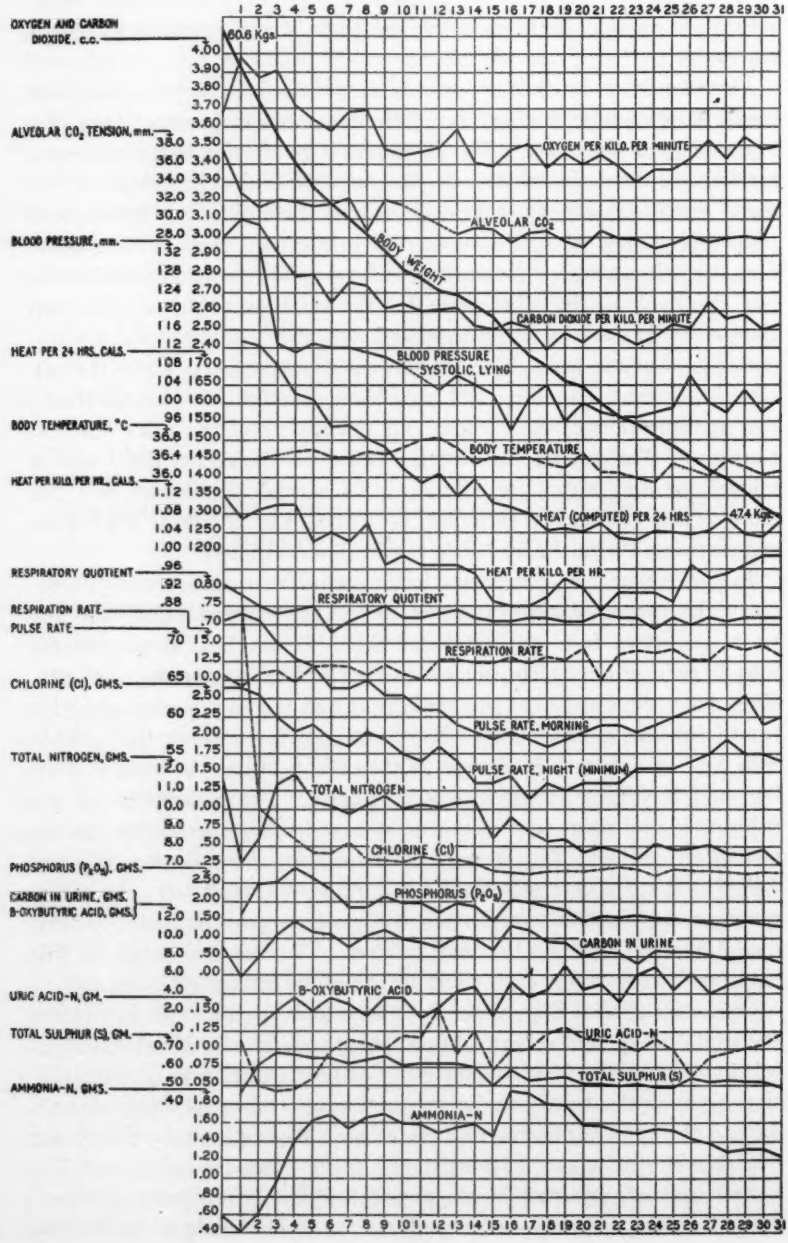
Measurements were made of the body-weight, insensible perspiration, rectal temperature fluctuations, pulse-rate, blood pressure, and the morphology of the blood. The mechanics of respiration and the alveolar air tension were also observed. A complete clinical examination was made every second day with a record of subjective impressions and psycho-physiological observations. The intestinal flora were studied and the skin excretion determined. A complete chemical examination was also made of the urine, including the partition of the nitrogen and the determinations of the chlorine, phosphorus, sulphur, total acidity, and B-oxybutyric acid. The mineral constituents, particularly the calcium, magnesium, sodium, and potassium excretion, were studied and the reducing power, the carbon, and the energy of the urine were determined. Throughout the fast the urine was examined daily with a microscope and tested for albumin. Of special significance was the study of the respiratory exchange, both with the universal respiration apparatus and with the large bed respiration calorimeter.

In the respiration calorimeter it was possible to measure simultaneously the carbon-dioxide excretion, the oxygen consumption, and the water vaporized from the lungs and skin. Direct heat measurements were likewise made which, when corrected for body temperature fluctuations, gave the exact heat production. From the results obtained with the respiration apparatus the effect of various factors upon the metabolism was studied, such as changes in the body position, the work of writing, and breathing oxygen-rich atmospheres. The metabolism of the subject during sleep was also compared with his metabolism in the waking condition. The energy transformation per kilogram of body weight and per square meter of body surface was computed. From the numerous respiration experiments with both apparatus and a careful record of daily activity, the total balance of income and outgo for this man for the thirty-one days could be computed with great accuracy.

Of special clinical importance are the records of the losses in weight; the studies of the rectal temperature, blood pressure and blood; the acidosis induced by the fast; and the effect of the fasting upon the composition of the urine and the respiratory exchange. The psychological studies and the general observations of the man showed that for thirty-one days the subject was able to exist in a fairly normal mental condition, entirely out of proportion to the physical decline in the body functions. The ill-advised insistence of the subject led to his breaking the fast by

METABOLISM CHART OF A MAN FASTING 31 DAYS

APRIL 14 - MAY 15, 1912



the selection of excessive amounts of acid fruit juices, thus inducing a most distressing colic and intestinal disorder; a short sojourn at a hospital was followed by convalescence.

It is impossible in the space available in these Proceedings to attempt an adequate summary of the results but the accompanying chart indicates the trend of some of the most important factors measured. The detailed results of the research are incorporated in a 416-page monograph issued as Publication No. 203 of the Carnegie Institution of Washington.

THE STUDY OF INDIAN MUSIC

By Alice C. Fletcher

PEABODY MUSEUM, HARVARD UNIVERSITY

Presented to the Academy, March 2, 1915

Thirty-five years of acquaintance with Indian music gathered from tribes of different linguistic families widely scattered over North America and a study, still in progress, of the music of a particular group has revealed facts relative to their music and its uses that possess an anthropologic value.

The term music as applied to the Indian refers solely to vocal music; for the natives of America possess few varieties of musical instruments beside the drum and rattle, both of which are used mainly to accent time and rhythm.

The number of Indian songs which have come to my personal attention number many hundreds. Those to which intensive study is being given have been secured from people classed as Plains Indians. All of these songs have been examined under their native conditions and in connection with the ceremonies, secular or religious, of which they were a part. When I began to observe and gather Indian songs, the graphophone was not available for field work. Securing songs by dictation was a difficult and unsatisfactory task for the reason that the Indians so frequently were averse to repeating the songs under observation, particularly when they were religious in character. This obstacle has been entirely overcome by the use of the graphophone, as one singing will give a record that can be repeated any number of times for the purposes of transcription, verification, and other study. For over twenty-five years I have used the graphophone when engaged in the field study of native ceremonies containing rituals and songs.

The word 'song' to our ears, suggests words arranged in metrical form and adapted to be 'set to music,' as we say. The native word

which is translated 'song' does not suggest any use of words. To the Indian, the music is of primal importance, words may or may not accompany the music. When words are used in a song, they are rarely employed as in a narrative, the sentences are not apt to be complete. In songs belonging to a religious ceremony the words are few and partake of a mnemonic character. They may refer to some symbol, may suggest the conception or the teaching the symbol stands for, rarely more than that. Vocables are frequently added to the word or words to eke out the musical measure. It sometimes happens that a song has no words at all, only vocables are used to float the voice. Whether vocables alone are used or used in connection with words, they are never a random collection of syllables. An examination of hundreds of songs shows, that the vocables used fall into classes; one class is used for songs denoting action, another class for songs of a contemplative character, and it is also noted that when once vocables are adapted to a song they are never changed but are treated as if they were actual words.

As Indian music is exclusively vocal, its range is confined to the compass of the human voice, which rarely exceeds three full octaves. This limited range, taken with its strictly vocal character, places Indian music in a class apart from the 'culture music' of our race. The remarkable development of 'culture music' has been due, in a large measure, to the invention and use of musical instruments. By means of these instruments the range of musical sounds has been increased far beyond that possible to the human voice and different qualities of tone have been secured. As a result, 'culture music' has been able to use diversity in musical sounds, to employ various melodic and harmonic forms and to acquire an objective and intellectual character quite impossible to the simple song from which it sprang. By the study of Indian music it is possible to retrace some of the steps that have led from song to 'culture music.' In pursuing this task, an important and helpful factor is found in certain conditions that have here obtained, namely: the natives of this continent, previous to the coming of our race, had not been subjected to inroads from alien peoples who might have disturbed the continuity of aboriginal culture; consequently the music and the life of the Indians can be observed in a simple, rather than a 'compound' environment.

Indian music presents several aspects to the student, all of which fall into two classes; the one technic, the other psychic in nature. The technic class has already received considerable attention from scholars, as to the tones used, the order in which they occur, the relation of this

native order to the formulated scales of 'culture music,' native musical form, melody, rhythm and the interrelation between different rhythms maintained by the voice and the drum or rattle.

From a present study of some of the psychic aspect of Indian music I have already reached certain conclusions, some of which are here presented.

Among the group of Indians whose music is under observation, the following customs, observed in some of the tribes, illustrate something of the fundamental character of the personal and emotional elements found in Indian song.

When at puberty the youth passes through the fasting rites introductory to the duties pertaining to manhood, 'the vision' that then appears to him, becomes his most sacred, personal experience, one that he never fully shares with another, and the cadence, or song, that generally accompanies the 'vision' remains in his memory and is to the man his secret personal appeal to the unseen powers he believes to have control over his life. This strain of music, constitutes his personal prayer, his cry for help in his hour of need. These 'songs' are strictly individual and emotional. They are rarely, if ever, heard by anyone save the singer.

Again: Certain societies require that each member have a special song, this song is generally of the man's own composition, although sometimes these songs are inherited from a father or a near relative who when living had been a member of the society. These individual songs are distinct from songs used in the ceremonies and regarded as the property of the society, although the members are entitled to sing them on certain occasions. When this society holds its formal meetings a part of the closing exercises consist of the simultaneous singing by all the members present of their individual songs. The result is most distressing to a listener, but there are no listeners, unless by chance an outsider is present, for each singer is absorbed in voicing his own special song which is strictly his personal affair, so that he pays no attention to his neighbour, consequently the pandemonium to which he contributes does not exist for him. Another phase of Indian song is here exhibited, but it is more directly connected with the Indian's manner of singing than with the music itself. This manner has tended to influence the estimation of outsiders of Indian song.

In a general way, an Indian singer makes no special effort, nor is he much concerned, to present his song in such a way as to give to the listener a musical picture. Practically no attempt is made to give what we call 'expression.' The song is apt to move along in strict time; any

change or break in the time or rhythm disrupts the flow of the music and, to the Indian, destroys the symmetry of the song. The Indian's manner of singing, his method of rendering his song is the outcome of the close emotional relation between the singer and the music. To a degree, it may be said that the Indian does not listen to his song as something objective to himself, yet he is found to be keenly aware of any slight change in the rhythm or tones of a melody—any mistake in the rendition of a song. In some societies a fine is imposed upon a member who makes mistakes in singing the songs. Years of experience has shown an unfailing demand for accuracy in the transcription and reproduction of a song. This demand betrays a consciousness and a recognition on the part of the Indian of musical form, both as to rhythm and the succession of tones that form a melody. In this consciousness is discerned one of the very early steps in the long path that leads to an artistic development of music.

Accuracy in the observance of religious rites arises from certain beliefs concerning the ability of these rites to open a way to the unseen powers, mistakes 'make the path crooked.' Indian ceremonies are ritualistic and symbolic objects are generally present. These objects have to be handled in peculiar and definite ways, with certain movements of the body and hands and accompanied by long recitations either chanted or intoned. Mistakes must be avoided as any carelessness is believed to be punished by supernatural means, therefore various mnemonic devices are employed by those who officiate, to insure against mistakes. In some tribes, if a mistake chances to be made in certain ceremonies, everything stops at the instant it is discovered. When the ceremony is resumed, it must start afresh, as though it had never been begun. Ceremonies that have elaborate rituals are generally attended only by the initiated and the rituals recited are known only to those who have paid the costly fees required from those who seek to be instructed in their use. In some ceremonies where lengthy intoned recitations occur these rituals are broken in upon by songs, sung by those officiating. These interpolated songs partake of the general character of Indian music and are emotional in their nature. The few words usually refer to the symbolic objects belonging to the ceremony,—the mind is thus directed toward the sacred object and the emotion evoked by the thought of it and its helpful relation to the people finds expression in the music. An old priest explained this introduction of songs into the long rituals, by saying: "The words talk to us, but harmonious sounds unite the people!" This explanation given by the old priest is confirmed by observations made as to the effect of songs pertaining to ceremonies having for

their purpose the stimulation, among the people taking part, of a common recognition, as of the gift of food, the value of tribal unity, of peace within or without the tribe. Many of the songs belonging to this class of ceremonies, although led by those who officiate in the rites, are sung by all present, men, women, and even children joining in these songs. The singing is always in unison, the personal emotion of each singer adds to the psychic force bred of a social accord, as all the people in one voice sing their thanks to the Unseen Giver of Life.

On a similar but lower plane are the songs belonging to vocations, by these not only are the hands strengthened for their task, but the unison singing of the song helps to development of social consciousness.

From what has been stated above it will be apparent that a wide field is opened by the study of Indian music not only concerning the beginnings of 'culture music' but in revealing the psychic influence of song upon the individual and upon his social development.

SOME RECENT ANTHROPOLOGICAL EXPLORATIONS

By Aleš Hrdlička

DIVISION OF PHYSICAL ANTHROPOLOGY, U. S. NATIONAL MUSEUM, WASHINGTON

Presented to the Academy, March 2, 1915

In 1912 an arrangement was entered into by the Smithsonian Institution and the Panama-California Exposition at San Diego, by which it became possible to send out jointly a number of anthropological expeditions for the purpose of furnishing the Exposition with original material for its exhibits, and of serving the Smithsonian Institution and the science of anthropology by advancing knowledge in directions in which progress was especially desirable.

The objects particularly aimed at in the latter respect were the promotion of research into man's antiquity; the survey of certain parts of northern and eastern Asia in quest of possible traces of racial connection with America; and the accumulation of reliable data on child development in certain primitive races under widely differing environments. Owing to illness in the personnel of the expeditions, to the European war, and other untoward circumstances, only a part of these plans could be fully carried out; but the results are of sufficient importance to encourage further activity in the directions named.

The several expeditions and their results, briefly outlined, were as follows:

Ancient Man in Europe and Asia. Field work was done by J. Matiegka, head of the Anthropological Bureau of the Bohemian University

at Prague, on the subject of neolithic and later prehistoric man in central Europe. Other explorations were carried on by K. Stolyhwo, head of the Anthropological Institute of Warsaw in relation to: (*a*) ancient man of southwestern Russia; (*b*) ancient man of the Yenesei Valley, Siberia. And a journey was made by me for the purpose of making a survey of the various well authenticated ancient skeletal remains of man in the Museums of Europe and of the sites of their discovery.

The direct results of these expeditions were the acquisition of a series of valuable specimens, including a trephined and mineralized neolithic skull in excellent state of preservation, and of valuable data which in part have already been published[1] and in part are in preparation for publication.

The kourgans or burial mounds in northeastern Russia were found to range in age from the neolithic to the Scythic periods. They yielded some exceedingly interesting skeletal material of man as well as that of various animals, some of which are now extinct in these regions. The Yenisei caves, regrettably, gave few skeletal remains, though such could doubtless be obtained by further exploration; but they yielded objects of neolithic culture, which may be of value in future archeological comparisons with the more eastern portions of Asia and possibly even with America. The examination of the ancient skeletal finds in Europe showed, besides a series of more or less doubtful specimens, a wealth of thoroughly authenticated material relating to man's descent, and substantiating his gradual evolution, during the Quarternary, from lower primate forms.

Search in Asia for Traces of the Race that Peopled America. Besides the explorations in the Yenisei caves, above mentioned, two trips were made to northeastern Asia. The first of these, made by me, extended to the upper Yenisei and Selenga river territories and to outer Mongolia, while the second, by St. Poniatowski, head of the recently established Ethnological Institute at Warsaw, penetrated to the living tribes in the Primorskaia Oblast, Siberia. The results of these expeditions comprise anthropological data, a valuable series of photographs, and a collection of rare skeletal material. The evidence indicates that there still exist throughout extensive areas of northern and eastern Asia remnants of former prehistoric (rather than 'paleoasiatic') peoples closely resembling the American Indian in type, mentality and other important particulars. In southern Siberia, Mongolia, and the regions to the eastward, there are also extensive archeological remains in the forms of mounds and occupation sites, ranging in chronology from neolithic to historic times. The exploration of these has as yet scarcely been com-

menced, and may be expected, when properly entered upon, to reveal details of the greatest consequence to American anthropology, for here somewhere must lie the remains, cultural and skeletal, of the ancestors of those who peopled the American continent.[2]

Comparative Study of Child Growth among Primitive Tribes. It was planned to conduct researches in this especially important field among the aboriginal Australians, the Negrito, the African Pygmies, the Zulu, the Eskimo, and the Chinese; but illness of the available explorers, the war in Europe, and other unforeseen conditions interfered with the proposed excursions to the Australians, Pygmies, and Chinese. The Negrito of the Philippines were studied by P. Newton, demonstrator in Anatomy of Georgetown University; and four hundred individuals, including many children, were measured. Particular care was taken to reach the full-bloods, which involved some perilous journeys. A series of photographs and some interesting skeletal remains were obtained, but casting became impossible, the dampness of the climate (the journey had to be undertaken partly during the rainy season) being such that no plaster could be preserved in dry condition. The Eskimo of St. Lawrence Island, Alaska, were examined by Riley D. Moore, aid in the Division of Physical Anthropology of the National Museum; 180 individuals, including a good proportion of children and adolescents, were measured; 30 facial casts and numerous photographs were made; and an important collection of skeletal material was gathered. The data secured and the collections from this island are of particular interest, as they relate to a group of people intermediary between the American and Asiatic Eskimo. Finally, an expedition, occupying fourteen months, was made by V. Shuck, an able anthropologist of Prague, to the Zulu, the Bushmen, and some British East Africa negroes, resulting in the acquirement of facial casts of 24 Zulus and 20 Bushmen, numerous photographs and other material, and anthropometric observations on upward of 800 children and adolescents of known age. The scientific data have already been partly utilized in charts for the Exposition, and are to be reported on as soon as conditions in Europe permit. They should form an excellent basis for eventual comparisons with observations of similar nature on the American negro, and also serve, of course, for the study of further contrasts or resemblances between the black and the white man. Duplicates of all the African measurements are preserved in the National Museum.

In addition to the above, the first four months of 1913 were spent by me on an expedition to Peru, the objects of which were to conduct a greatly needed extension of former anthropological explorations in that

country, and to obtain a collection of material for illustrating prehistoric aboriginal pathology and surgery. Considerable success was met with in both directions. The anthropology of the coast was mapped out for the distance of approximately 600 miles, and some insight was obtained into that of the highlands. It was ascertained that important separate political and cultural coastal groups, such as the Chimu and the Nasca people, were no special units, anthropologically, but belonged to the same physical type as the rest of the coast population. The collections made on this trip, being selections from nearly 5000 burials, are especially valuable. Finally, the exploration made possible rich original exhibits at San Diego, covering practically the whole field of pre-Columbian Indian pathology, to which are added 60 crania showing all the forms of ancient Indian trephining. The general results of this expedition have already been published,[3] but the material collected offers a rich opportunity for further investigation.

[1] Hrdlička, A., The most ancient skeletal remains of man, *Smithsonian Rept.* for 1914, pp. 491–552, pls. i–xli.

[2] For preliminary reports on this work, see *Smithsonian Inst., Misc. Collect.*, 60 (1912); *Compte-Rendu XIV Cong. Intern. d'Anthropologie et d'Archéologie Préhist.*, Genève, 1913; and *Trans. XVIII Intern. Cong. Americanists*, London, 1914.

[3] Hrdlička, A., Anthropological work in Peru in 1913, with notes on the pathology of the ancient Peruvians. *Smithsonian Inst., Misc. Collect.*, 61, no. 18 (Publication 2264), 1914, pp. i–v, 1–69, 26 pls.

THE SECOND DERIVATIVES OF THE EXTREMAL INTEGRAL FOR A GENERAL CLASS OF PROBLEMS OF THE CALCULUS OF VARIATIONS

By Arnold Dresden

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN

Presented to the Academy, January 18, 1915

1. In an earlier paper,[1] I obtained expressions for the second derivatives of the extremal-integral for the problem of minimizing the integral

$$\int F(x, y, x', y')dt, \tag{1}$$

in terms of fundamental solutions of the Weierstrass form of Jacobi's differential equation for that problem.[2] In the same paper these expressions were used for deriving necessary conditions which must be satisfied by a curve which is to minimize the integral (1), if one or both endpoints are allowed to vary along a curve,[3] or if curves are admitted whose slopes possess a finite number of finite discontinuities.[4]

2. The method of differentiation employed in arriving at these re-

sults had been used before, probably first by Poisson or by Jacobi,[5] in the treatment of the integral (1) and also of more general integrals. These uses had been restricted however either to the derivation of conditions of the first order, or else to the study of special problems of variable endpoints.[6] Now it is evident that the number of special problems of this character increases rapidly as one proceeds to consider problems involving more and more unknown functions. To secure a general mode of treatment for all these special cases, it has seemed desirable to secure formulas for the second derivatives of the extremal-integral in more general problems than the one treated in my paper cited above.

3. The present paper derives such formulae for the integrals

$$\int f(x, y_1, \cdots, y_n; y_1' \cdots, y_n')\, dx, \quad \text{where } y' = dy/dx, \tag{2}$$

and

$$\int F(y_1, \cdots, y_n; y_1', \cdots, y_n')\, dt, \quad \text{where } y' = dy/dt. \tag{3}$$

In the case of the integral (2) the classical theory suffices to carry the work through and one secures expressions for the second derivatives of the extremal-integral in terms of two sets of conjugate solutions of the self-adjoint system of Jacobi differential equations:

Denoting by u_{jk} and v_{jk} systems of solutions of Jacobi's differential equations, which satisfy the initial conditions

$$u_{jk}(x_1) = \delta_{jk},\ u_{jk}(x_2) = 0;\ v_{jk}(x_1) = 0,\ v_{jk}(x_2) = \delta_{jk},$$

and using the current abbreviated notation:

$$f_0 = \frac{\partial f}{\partial x},\ f_{n+i} = \frac{\partial f}{\partial y_i'},\ \text{etc.};\ f_0^{(r)} = f^0(x_r),\ \text{etc.};\ y'_{ri} = y'_i(x_r) = \frac{d}{dx}\, y_i(x)\Big|_{x=x_r},$$

we have

$$\frac{\partial^2 I}{\partial x_1\, \partial x_2} = -\sum_{ijk} f^{(1)}_{n+i,\, n+j}\, y'_{1i}\, y'_{2k}\, v'_{kj}(x_1) = \sum_{ijk} f^{(2)}_{n+i,\, n+j}\, y'_{2i}\, y'_{1k}\, u'_{kj}(x_2),$$

$$\frac{\partial^2 I}{\partial x_r^2} = (-1)^r\Big[f_0^{(r)} - \sum_i f^{(r)}_{0,\, n+i}\, y'_{ri} - \sum_{ij} f^{(r)}_{n+i,\, n+j}\, y'_{ri}\, y''_{rj} + \sum_{ijk} f^{(r)}_{n+i,\, n+j}\, y'_{ri}\, y'_{rk}\big[\delta_{r1} u'_{kj}(x_r) + \delta_{r2} v'_{kj}(x_r)\big]\Big],$$

$$\frac{\partial^2 I}{\partial x_r\, \partial y_{ri}} = (-1)^r\Big[f_i^{(r)} - \sum_j f^{(r)}_{n+j,\, i}\, y'_{rj} - \sum_{jk} f^{(r)}_{n+j,\, n+k}\, y'_{rj}\big[\delta_{r1} u'_{ik}(x_r) + \delta_{r2} v'_{ik}(x_r)\big]\Big]$$

$$= (-1)^r\Big[f^{(r)}_{0,\, n+i} + \sum_j f^{(r)}_{n+i,\, n+j}\, y''_{rj} - \sum_{jk} f^{(r)}_{n+i,\, n+j}\, y'_{rk}\big[\delta_{r1} u'_{kj}(x_r) + \delta_{r2} v'_{kj}(x_r)\big]\Big],$$

$$\frac{\partial^2 I}{\partial x_r\, \partial y_{si}} = (-1)^{r-1} \sum_{jk} f^{(r)}_{n+j,\, n+k}\, y'_{rj}\big[\delta_{r1} v'_{ik}(x_r) + \delta_{r2} u'_{ik}(x_r)\big]$$

$$= (-1)^r \sum_{jk} f^{(s)}_{n+i,\, n+j}\, y'_{rk}\big[\delta_{r1} u'_{kj}(x_s) + \delta_{r2} v'_{kj}(x_s)\big],$$

$$\frac{\partial^2 I}{\partial y_{ri}\,\partial y_{rj}} = (-1)^r\Big[f^{(r)}_{n+i,\,j} + \sum_k f^{(r)}_{n+i,\,n+k}\,[\delta_{r1}u'_{jk}(x_r) + \delta_{r2}v'_{jk}(x_r)]\Big],$$

$$\frac{\partial^2 I}{\partial y_{ri}\,\partial y_{sj}} = (-1)^r \sum_k f^{(r)}_{n+i,\,n+k}\,[\delta_{r1}v'_{jk}(x_r) + \delta_{r2}u'_{jk}(x_r)],$$

where $i, j, k = 1, \ldots, n$; $r = 1, 2$; $r + s = 3$; and $\delta_{jk}\begin{cases} = 1 \text{ when } j = k \\ = 0 \text{ when } j \neq k. \end{cases}$

4. For the integral (3), there is not in the literature a theory of the second variation and of Jacobi's equation analogous to Weierstrass's work for the integral (1). As an aid towards securing such a theory, the author established conditions under which it is possible to reduce a set of n linearly independent linear differential forms in $n + 1$ functions to a set of n such forms of the same order in n linear combinations of these $n + 1$ functions.[7] This reduction is now used to carry through a reduction of the second variation and of Jacobi's differential equations for the integral (3), analogous to Weierstrass's reduction for the integral (1). This reduction is dependent upon the non-vanishing of one of the functions y'_i. In every regular problem one is assured that there is at every point of the interval (t_1t_2) a function y_i, whose derivative does not vanish at the point, but the function may be a different one at different points. By the use of the Heine-Borel theorem, we can then, in virtue of the continuity conditions on the functions y_i, divide the interval (t_1t_2) in a finite number of intervals in each of which the reduction is possible. We consider now the system of reduced Jacobi equations which exists in the neighborhood of the point $t = t_0$ and express the second derivatives of our extremal-integral in terms of sets of fundamental solutions of these equations. These solutions are known to exist throughout the entire interval (t_1t_2), even though the equations which they satisfy may not be valid beyond a sub-interval. In this manner a set of formulae is secured, which may be immediately applied for finding analytical conditions for a minimum of the integral (3), when one of both endpoints are allowed to vary in manifolds of $1, 2 \ldots, n - 1$ dimensions.

It is intended further to develop similar formulas for the problems of minimizing integrals (2) and (3) if the unknown functions are further conditioned so as to satisfy a system of differential equations, or a system consisting of differential and of algebraic equations.

[1] A. Dresden, *Trans. Amer. Math. Soc.*, 9, 467 (1908).

[2] See Bolza, *Vorlesungen über Variationsrechnung*, p. 233 and p. 312.

[3] Ibid., p. 316–318 and p. 328–330. These conditions had been known before.

[4] Ibid., p. 372–389.

[5] See Dienger, *Grundriss der Variationsrechnung*, p. 27.

[6] See Mayer, *Leipzig, Ber. Ges. Wiss.*, **36**, 99 (1884) and **48**, 436 (1896); Bliss, *Math. Ann., Leipzig*, **58**, 70 (1903); Erdmann, *Zs. Math., Leipzig*, **23**, 364 (1878).

[7] "On the reduction of a system of linear differential forms of any order," *Annals Mathematics*, **13**, 149 (1912). In a paper entitled On the second variation, Jacobi's equation and Jacobi's theorem, *Ibid.*, **15**, 78 (1913), this reduction was used to unify the Weierstrass theory of the second variation and Jacobi's equation.

GROUPS POSSESSING AT LEAST ONE SET OF INDEPENDENT GENERATORS COMPOSED OF AS MANY OPERATORS AS THERE ARE PRIME FACTORS IN THE ORDER OF THE GROUP

By G. A. Miller

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS

Presented to the Academy, January 18, 1915

If $S_1, S_2, \ldots, S_\lambda$ represent a set of operators of the group G such that these λ operators generate G but that no $\lambda-1$ of them generate G, then these λ operators are called a set of independent generators of G. Every subset of a set of independent generators of any group whatever generates a subgroup whose order contains at least as many prime factors as the number of operators in this subset. In particular the number of operators in a set of independent generators never exceeds the number of prime factors in the order of the group except in the trivial case when the group is the identity, unity not being regarded as a prime factor.

If G represents the abelian group of order, p^m and of type $(1, 1, 1, \ldots)$, it is evident that each of its possible sets of independent generators involves as many operators as there are prime factors in the order of G. In what follows we propose to determine some properties possessed by all those groups which have at least one set of independent generators composed of as many operators as there are prime factors in the order of the group. The symbol G will hereafter in the present article be used to represent any one of these groups, and we shall assume in what follows that $S_1, S_2, \ldots, S_\lambda$ always represent a set of independent generators of G such that λ is equal to the number of the prime factors of the order g of G.

Each of the operators $S_1, S_2, \ldots, S_\lambda$ must be of prime order since each subset of these independent generators generates a subgroup whose order has exactly as many prime factors as the number of operators in this subset. A necessary and sufficient condition that the groups generated by two such subsets have only the identity in common

is that these two subsets have no common operator. If two of the operators $S_1, S_2, \ldots, S_\lambda$ are of different orders, the larger operator is transformed into a power of itself by the smaller, since these two operators generate a group of order pq, p and q being distinct prime numbers. In particular, if the operators $S_1, S_2, \ldots, S_\lambda$ are not all of the same order those of highest order generate an invariant subgroup of G. If these orders represent more than two prime numbers, the operators of highest order together with those of the next lower order generate an invariant subgroup of G, etc.

If the set of independent generators $S_1, S_2, \ldots, S_\lambda$ contains at least two non-commutating operators of the same order p, let S_1, represent one of these operators which is commutative with the smallest number of the other operators of order p in the given set; and let $S_2, \ldots, S_\gamma$ represent all the operators of order p in the set $S_1, S_2, \ldots, S_\lambda$, which are not commutative with S_1. We proceed to determine the group generater by $S_1, S_2, \ldots, S_\gamma$. Since S_1 and S_2 are non-commutative they generate a group of order p_1p, p_1 being a prime $>p$, which contains p_1 conjugate subgroups of order p. When $\gamma>2$, S_1 and S_3 will generate a similar group of order p_2p. It may be assumed that $p_1 \geqq p_2$.

The group generated by S_1, S_2 and S_3 is of order p_1p_2p. If $p_1=p_2$ this group contains a single subgroup of order p_1^2, and each of its remaining operators is of order p since it contains no invariant operator besides the identity. If $p_1>p_2$, the group in question contains a cyclic subgroup of order p_1p_2 since the quotient group with respect to the invariant subgroup of order p_1, cannot be cyclic. (See O. Hölder, *Göttingen Nachrichten*, 1895, p. 298.) Hence the group generated by S_1, S_2, and S_3 must always contain an abelian subgroup of index p whose order is prime to p, and each of its operators which is not contained in this subgroup must be of order p.

When $\gamma>3$, we may consider separately the groups generated by S_1, S_2, S_4 and by S_1, S_3, S_4. The operators of order p_2 in the group of order p_2p generated by S_1 and S_4 must therefore be commutative with the operators of orders p_1 and p_2 contained in the group generated by S_1, S_2 and S_3. Hence it results that S_1, S_2, S_3 and S_4 must generate a group having a single abelian subgroup of index p while all its remaining operators are of order p. As this mode of reasoning may clearly be continued until all the operators of the set $S_1, S_2, \ldots, S_\gamma$ have been exhausted, it results that these operators generate a group H having an abelian subgroup of index p whose order is prime to p, and that each of the other operators of H is of order p. In other words, H involves no invariant operator besides the identity, and the prime factors of the order of its subgroup of index p are all of the form $1 + kp$.

Since S_1 was so selected as to be commutative with the smallest possible number of operators of order p in the set $S_1, S_2, \ldots, S_\lambda$ it results that each of the operators $S_1, S_2, \ldots, S_\gamma$ is commutative with every other operator of order p in this set. Each of the operators of H whose order exceeds p must be commutative with every one of the independent generators $S_{\gamma+1}, \ldots, S_\lambda$. In fact, if the order of such an independent generator exceeds p, it must be transformed into a power of itself by all the operators of H, since it is transformed into a power of itself by each of the operators $S_1, S_2, \ldots, S_\gamma$, which generate H. It results therefore from the theorem of Hölder, to which we referred above, that the independent generator in question is commutative with every operator of H whose order exceeds p.

On the other hand, every operator of the set $S_1, S_2, \ldots, S_\lambda$ whose order is less than p, must transform each of the operators $S_1, \ldots, S_\gamma$ into a power of itself. This must be the first power for all of these operators. In fact, if S_α, $1 \leqq \alpha \leqq \gamma$, were not transformed into its first power by such an operator S_β then the group generated by S_α and some other operator of the set $S_1, \ldots, S_\gamma$ would be transformed into itself by S_β. This is clearly impossible in view of the Hölder theorem to which we referred above. Hence it results that every operator of H is commutative with every operator of the group generated by those operators of the set $S_{\gamma+1}, \ldots, S_\lambda$ whose orders do not exceed p.

Some of these results may be expressed in the form of a theorem as follows: *If a set of independent generators, which involves as many operators as there are prime factors in the order of G, includes at least two non-commutative operators of the same order p then all the operators of order p in this set, which are non-commutative with some one of them, generate a group H involving an abelian invariant subgroup of index p under H, and every operator of this invariant subgroup is commutative with every operator of the group generated by all the operators of the given set of independent generators of G, which are not contained in H.* It results also directly from the preceding developments that if H does not include all the operators of order p in the given set of independent generators of G, then these remaining operators of order p may be combined into subsets such that all the operators of each subset generate either an abelian group of order p^m and of type $(1, 1, 1, \ldots)$, or a group having the properties which we proved to apply to H. Hence the theorem.

If a group G contains a set of independent generators composed of as many operators as there are prime factors in the order of G then all the operators of the same order p in this set generate one of the following three groups: (1) *Abelian groups of order* p^m *and of type* $(1, 1, 1, \ldots)$; (2) *Non-abelian group having an abelian invariant subgroup of index p but*

of order prime to p, while all its other operators are of order p; (3) *Group which is the direct product of such groups as are defined in* (1) *and* (2).

When no operator in the given set of independent generators of G has an order which exceeds p, it results that the group generated by all the operators of order p in this set is invariant under G. When this set contains non-commutative operators of order p all of these operators which are non-commutative with some one of them generate an invariant subgroup, composed of operators which are commutative with every operator in the group generated by all the operators of the set except those used to generate this invariant subgroup. As an important but very special result we may mention the theorem that *every group which contains a set of independent generators composed of as many operators as there are prime factors in the order of the group is solvable.*

Since each operator in the abelian invariant subgroup of H is commutative with each of the operators of the set $S_1, S_2, \ldots, S_\lambda$, except those which are contained in H, it is easy to see that every Sylow subgroup of G is abelian and of type $(1, 1, 1, \ldots)$. In fact, if all the operators of the same order in the set $S_1, S_2, \ldots, S_\lambda$ are commutative the various subsets composed of the operators of the same order in this set would generate such Sylow subgroups of G. If not all the operators of the same order in $S_1, S_2, \ldots, S_\lambda$ are commutative then these non-commutative operators may be divided into subsets such that each subset generates a group having the properties which have been proved to belong to H. Hence the theorem: *If a group contains at least one set of independent generators composed of as many operators as there are prime factors in the order of the group then all of its Sylow subgroups are abelian and of type* $(1, 1, 1, \ldots)$. As a special case of this theorem we have the known and evident result that when the order of such a group is a power of a prime the group must be abelian and of type $(1, 1, 1, \ldots)$.

Among the important categories of groups which have the property that it is possible to find a set of independent generators composed of as many operators as there are prime factors in the order of the group is the important category of solvable groups composed of all the groups whose order is not divisible by the square of a prime number. If the order of such a group involves more than one prime factor, it is always possible to select a set of independent generators composed of an arbitrary number of operators from 2 to the number of prime factors in the order of the group. This fact can be deduced directly from the theorem due to Hölder to which we referred above. This important category of solvable groups is therefore included in the larger category of such groups considered above.

POINT SETS AND ALLIED CREMONA GROUPS

By Arthur B. Coble

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY

Presented to the Academy, January 18, 1915

It is my purpose to develop, in a series of papers of which the first has been submitted to the *Transactions of the American Mathematical Society*, some aspects of the theory of an ordered set of n discrete points in a linear projective space, S_k, of k dimensions.[1] Such a set, denoted by P_n^k, defines projectively an 'associated' set Q_n^{n-k-2} in S_{n-k-2}, the two sets being mutually related. The study of this type of association, begun by Rosanes and Sturm,[2] is continued here. If $n = 2k + 2$ the associated sets can lie in the same S_k, and in particular they may coincide in order forming thus a 'self-associated' set. In addition to projective constructions for associated and self-associated sets, certain irrational conditions for such types of association which seem to be novel are given.

The invariants of a set P_n^k are formed from the determinants of the matrix of the coördinates of the points. An invariant is required to be homogeneous and of the same degree in the coördinates of each point and to be unaltered under permutations of the points. It appears that associated sets have proportional invariants. This theory of invariants is the natural extension of the theory of binary forms if the form be regarded as the definition of a point set P_n^1. Complete systems for P_6^1 and P_6^2 are derived and the accompanying algebra is utilized to obtain explicit equations not only for the cubic surface mapped from the plane by cubic curves on P_6^2 but also for the tritangent planes and the lines of the surface.

By a certain construction the ordered set P_n^k is mapped upon a point P of a space $\Sigma_{k(n-k-2)}$. The Cremona transformation from one map P to another map P' of the same set is discussed and eventually the construction is simplified so that this transformation is linear. The $n!$ permutations of the points of P_n^k lead to $n!$ points P in Σ which are conjugate under a Cremona group $G_{n!}$ in Σ. When $k = 1$, $G_{n!}$ is the 'cross-ratio group' of Moore.[3] A set of generators of $G_{n!}$ is exhibited. The invariant spreads of $G_{n!}$ are obtained from the invariants of P_n^k. The linear system of spreads of lowest order invariant under $G_{n!}$ is derived from the irrational invariants of P_n^k of lowest weight. Associated sets determine the same group.

The form-problem of the $G_{5!}$ determined by the associated P_5^1 and P_5^2 leads to a solution of the quintic equation; that of the $G_{6!}$ determined

by the associated P_6^1 and P_6^3 and by the self-associated P_6^2 to a solution of the sextic equation. These equations have been discussed from this point of view in earlier papers of the writer.[4] The principal novelty here is the use of a connection between the self-associated P_6^2 and the theta modular functions of genus two to solve the related form-problem. In general one may say that the theory of the point set furnishes a common algebraic background not only for the general equations of degree n but also, as will appear later, for such particular equations as are involved in the determination of the lines on a cubic surface and of the double tangents of a plane quartic curve.

By introducing certain conventions it is possible to establish a mutual order for the two sets of singular points of a Cremona transformation in the plane. Since ordinary corresponding point pairs also are mutually ordered the addition of any number of ordinary pairs to the sets of singular points leads to a pair of sets which are said to be 'congruent' under Cremona transformation. The conditions under which congruence can occur are fundamentally important for effective application of the Cremona transformation to geometry or analysis. These conditions are derived up to a certain natural limit. Since two sets congruent to a third are congruent to each other the projectively distinct sets in the plane which are congruent in any order with a given set P_n^2 will be mapped by an aggregate of distinct points in Σ such that the aggregate is determined by any one of its points. If the type of transformation under which the congruence occurs and if the order be given the set is uniquely determined so that the transition from one point in Σ to another is effected by a Cremona transformation. In other words the sets P_n^2 congruent in any order with a given P_n^2 are mapped in Σ by points conjugate under an 'extended' Cremona group $G_{n,2}$ which contains G_{n1} as a subgroup. For $n = 6, 7, 8$ the order of $G_{n,2}$ is $6!.72$, $7!.288$, $8!.2^6.135$ respectively but for larger values of n the $G_{n,2}$ is infinite and discontinuous. The finite groups are isomorphic respectively with those of the lines on a cubic surface, the bitangents of a plane quartic curve, and the tritangent planes of a space sextic of genus four on a quadric cone.

Let us call a Cremona transformation in S_n which can be expressed as a product of the particular transformations obtained by inverting the variables a 'regular' transformation. Obviously the regular transformations are the elements of a 'regular group.' Then the definition of congruence of point sets in S_2 as given above can be generalized to include congruence under regular transformations in S_n. Again the set P_n^k defines as above an extended group $G_{n,k}$ of Cremona transfor-

mations in $\Sigma_{k(n-k-2)}$ which in general is infinite and discontinuous. Associated sets define the same group. The only finite types in this doubly infinite (for increasing n and k) series of groups are those defined by the sets mentioned above or by their associated sets. Each $G_{n,k}$ contains $G_{n-1,k}$ as a subgroup.

The Cremona transformations in S_k under which the congruence of sets P_n^k occurs will have a certain effect upon the spreads in S_k. If the order of the spread and its multiplicity at each point of P_n^k be taken as variables,[5] this effect is expressed by a linear transformation on $n+1$ variables. Thus there is determined a group of linear transformations $g_{n,k}$ in S'_n with integer coefficients which is isomorphic with the Cremona $G_{n,k}$. In S'_n, the $g_{n,k}$ has an invariant S'_{n-1} within which it determines an isomorphic $g'_{n,k}$ of linear transformations whose coefficients are rational numbers. The importance of the finite types in these series of groups would seem to indicate that interesting applications of the infinite types may be expected.

A definite application of the foregoing theory to the determination of the lines on a general cubic surface K^3 will be made.[6] As has been mentioned earlier, cubic curves on P_6^2 map S_2 upon a general cubic surface which appears in Cremona's hexahedral form, C^3. The rational invariants of P_6^2 are rational invariants of the surface after the adjunction of the irrationality which isolates a set of six skew lines of a double six. Such invariants determine, and are determined by, the invariant spreads of $G_{6!}$ in Σ_4. If it be required further that the spreads be invariant under the extended group, $G_{6,2}$ of order 51840 in Σ_4 they determine the invariants of C^3 itself. In this way not only the invariants but also the linear covariants of C^3 are calculated and identified with those of K^3. Now K^3 being given, its invariants and linear covariants are known. The invariants are the known quantities in the form-problem of $G_{6,2}$. The solution of this form-problem furnishes a point P in Σ_4, the map of P_6^2. From P_6^2 as indicated earlier the lines of C^3 as well as the linear covariants of C^3 are derived. The linear transformation obtained from the identification of the linear covariants of C^3 and K^3 transforms the known lines of C^3 into the required lines of K^3. All of the processes outlined here are rational except the solution of the form-problem of $G_{6,2}$. This may be dismissed as a problem in the theory of functions or it may by means of an accessory irrationality be reduced to the form-problem which occurs in the trisection of the periods of the hyperelliptic functions of genus two. The ultimate importance of this latter problem has been pointed out by Klein.[7] It would be more desirable however to utilize directly the theta-modular

functions in the solution of the problem of $G_{6,2}$ as has been done in the case of the sextic.

The developments sketched above suggest a number of inquiries which promise results of interest in various directions. Also some of the problems considered such as that of the invariants of P_n^k are worthy of closer study than has been given them in the account here reported.

[1] This investigation has been pursued under the auspices of the Carnegie Institution of Washington, D. C.

[2] For references see Study, *Math. Ann., Leipzig*, **60**, 348 (footnote).

[3] E. H. Moore, *Amer. J. Math.*, **22**, 279 (1900).

[4] *Trans. Amer. Math. Soc.*, **9**, 396 (1908), and **12**, 311 (1911). The relation of these papers to the earlier work of Klein and others is there set forth at length.

[5] S. Kantor has used this device for an S_2 in his crowned memoir: *Premiers Fondements pour une Théorie des Transformations Périodiques Univoques*, Naples (De Rubertis), 1891.

[6] This will appear as a joint paper by Mr. C. P. Sousley and A. B. Coble.

[7] Letter to Hermite, *J. Math., Paris*, Ser. 4, **4**, 169. See also *Math. Ann., Leipzig*, Witting, **29**, 167; Maschke, **33**, 317, **36**, 190; and Burkhardt, **38**, 161.

THE STRAIGHT LINES ON MODULAR CUBIC SURFACES

By L. E. Dickson

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO

Presented to the Academy, January 21, 1915

1. In ordinary space a cubic surface without singular points contains exactly 27 straight lines, of which 27, 15, 7, or 3 are real; there are 45 sets of three coplanar lines, the three of no set being concurrent. In modular space, in which the coördinates of points and the coefficients of the equations of lines or surfaces are integers or Galois imaginaries taken modulo 2, it is interesting to notice that *three coplanar lines on a cubic surface may be concurrent* (§2). A point with integral coördinates is called real. A line or surface is called real if the coefficients of its equations are integers. In space with modulus 2, the number of real straight lines on a cubic surface without singular points is 15, 9, 5, 3, 2, 1, or 0.

We shall give here an elementary, self-contained, investigation of some of the most interesting cubic surfaces modulo 2. A complete classification of all such surfaces under real linear transformation will appear in the *Annals of Mathematics*, but without the present investigation of the configuration of their lines.

2. Every real point of space modulo 2 is on the surface[1]

$$xy(x + y) = zw(z + w). \tag{1}$$

since each member is an even integer when x, y, z, w are integers.

Theorem 1. *This surface with exactly 15 real points contains exactly 15 real straight lines. There are exactly 15 sets of three coplanar real lines on the surface, the three of each set being concurrent, while their plane is the tangent to the surface at the point of concurrence.*

The surface has the automorphs[2] (i.e., transformations carrying the surface into itself)

$$(zw);\ \pi = (xz)(yw);\ \beta\colon z^1 = z + w;$$
$$\gamma\colon x^1 = x,\ y^1 = y + w,\ z^1 = z + x,\ w^1 = x + z + w.$$

The fifteen real points of space

1 = (1000)	5 = (1100)	9 = (0101)	13 = (1011)
2 = (0100)	6 = (1010)	10 = (0011)	14 = (0111)
3 = (0010)	7 = (1001)	11 = (1110)	15 = (1111)
4 = (0001)	8 = (0110)	12 = (1101)	

are permuted transitively by these automorphs, since

$$(zw)\colon (3\ 4)\ (6\ 7)\ (8\ 9)\ (11\ 12),$$
$$\beta\colon (4\ 10)\ (7\ 13)\ (9\ 14)\ (12\ 15),$$
$$\pi\colon (1\ 3)\ (2\ 4)\ (5\ 10)\ (7\ 8)\ (11\ 13)\ (12\ 14),$$
$$\gamma\colon (1\ 13\ 12\ 6)\ (3\ 10\ 8\ 14)\ (4\ 9)\ (5\ 15\ 7\ 11).$$

The tangents to (1) at the fifteen real points give all of the fifteen real planes in space, since the tangent at $(XYZW)$ is

$$Y^2x + X^2y + W^2z + Z^2w = 0.$$

Hence these planes are permuted transitively by the real linear automorphs of the surface. The plane $x = 0$, tangent at 2, contains the concurrent lines

$$A\colon x = z = 0, \qquad B\colon x = w = 0, \qquad C\colon x = 0,\ z = w,$$

evidently on the surface. Hence each of the fifteen real planes contains three (concurrent) lines lying on the surface (1). The only real planes through line A are evidently $x = 0$, $z = 0$, and $x = z$. Hence the number of real lines on (1) is 15.3/3. We have therefore proved Theorem 1.

Since (zw) and β replace A by B and C and since the fifteen real planes are permuted transitively by the automorphs, the fifteen real lines on the surface are permuted transitively by the real linear automorphs.

We next make use of the imaginary automorph I which leaves z and w unaltered, but multiplies x and y by t, where $t^2+t+1 = 0$. It replaces the line $w = x$, $y = z$ on (1) by E_{t_0} $(w = tx,\ z = ty)$, where

$$E_{ab}\colon w = ax,\ y = bx + a^2z;\ D_{abc}\colon z = ax + bw,\ y = cx + a^2w.$$

The twelve imaginary lines in which b and c are integers, while $a^2 + a + 1 = 0$, are seen to be permuted transitively by (zw), π, β. We now have $15 + 12$ lines on (1). As in the algebraic theory, there are only 27 lines on a cubic surface without singular points.[3] Hence the 27 lines are permuted transitively by I and the real automorphs. To prove Theorem 2 below, it therefore suffices to take A as one of the three coplanar lines. The only planes through A are $z = 0$ and $x = kz$. The former cuts the surface in the lines A, $z = y = 0$, and $z = 0$, $x = y$, which concur at 4. The latter cuts the surface in the lines A and

$$x = kz, \ ky(y + kz) + w(z + w) = 0.$$

This quadratic function has the factor $w + \alpha y + \beta z$ if and only if $\beta^2 = \beta$, $k = \alpha^2$, $\alpha = k^2$. Hence A meets just 10 lines on the surface, the above two and

$$L_{k\beta}: \ x = kz, w = k^2y + \beta z \qquad (\beta = 0, 1; \ k^4 = k).$$

The only coplanar sets of three lines, one of which is A, are the above set of three real and the four sets A, L_{k0}, L_{k1}, which meet at $(010k^2)$. In all there are $27.5/3 = 45$ sets of coplanar lines.

Theorem 2. *The 27 straight lines on (1) form 45 sets of three coplanar lines. The three of each set concur and their plane is the tangent to the surface at the point of concurrence.*

3. There are several types of real cubic surfaces the numbers of whose real points and real lines differ from those of (1), but are derivable from (1) by imaginary linear transformation and hence have 45 sets of three coplanar concurrent straight lines.

If in (1) we replace z by $ez + e^2w$ and w by $e^2z + e^4w$, where $e^3 + e + 1 = 0$, and hence replace $z + w$ by $e^4z + ew$, we evidently get

$$xy(x + y) = z^3 + zw^2 + w^3. \tag{2}$$

For integral values of the variables, the left number is zero modulo 2, whence $z \equiv w \equiv 0$. Thus 1, 2, 5 are the only real points on (2); it contains no real line.

If in (1) we make also the like replacement of x, y, we get

$$x^3 + xy^2 + y^3 = z^3 + zw^2 + w^3. \tag{3}$$

It contains just nine real points and only three real lines: $z = x$, $y = w$; $z = y = x + w$; $w = x = y + z$, no two of which intersect.

If in (1) we replace[4] z by $z + bw$ and w by $z + b^2w$, where $b^2 + b + 1 = 0$, we get

$$xy(x + y) = w(z^2 + zw + w^2). \tag{4}$$

The only real points on it are evidently the seven in the plane $w = 0$. Hence the only real lines on it are the three having $w = 0$.

If in (2) we replace x by $x + by$ and y by $x + b^2y$, where $b^2 + b + 1 = 0$, we get

$$y(x^2 + xy + y^2) = z^3 + zw^2 + w^3. \tag{5}$$

Replacing y by $y + z + w$, z by $z + y$, w by $w + y$, we get a surface whose real points are those on the cone $yz + yw + zw = 0$ and hence are 1, 2, 3, 4, 5, 6, 7. Thus the real points are those on the lines $y = z = 0$, $y = w = 0$, $z = w = 0$, meeting at 1.

If in (1), written in Capitals, we set

$$X = x + y, \quad Y = bx + b^2y + (a^2 + 1)w, \quad Z = ax + (a + 1)y + z,$$
$$W = w, \quad b^2 + b + 1 \equiv 0, a^4 + a + 1 \equiv 0,$$

we get the real surface[5]

$$x^3 + y^3 + (x^2 + z^2)w + (y + z)w^2 = 0, \tag{6}$$

which has only eleven real points and only five real lines

$$w = 0, y = x; \qquad z = x + tw, y = x; \quad z = tw, y = x + w \quad (t = 0, 1).$$

4. A surface without singular points and not having every set of three coplanar lines concurrent, as was the case with all the preceding surfaces, is

$$xy(x + y) = w(w + z)(y + z). \tag{7}$$

It contains thirteen real points, 9 and 12 alone being not on it. We obtain a straight line on (7) by equating to zero one factor of each member. The resulting nine lines will be shown later to be the only real lines on (7). The three in $x = 0$ are not concurrent, likewise the three in $x = y$, while the three in $y = 0$ meet at 1. We obtain a redistribution of these nine lines by beginning with a factor of the second member of (7): the three lines in $w = 0$ meet at 3, the three in $w = z$ meet at 10, the three in $y = z$ meet at 4. There is no plane other than these six which contains three coplaner lines from this set of nine real lines.

Evident automorphs[6] of (7) are

$$(A) \quad z^1 = z + y + w; \qquad (B) \quad w^1 = w + z; \qquad (C) \quad x^1 = x + y.$$

Evident imaginary lines on (7) are

$$(H_a) \quad w = ax, z = ay \qquad (a^3 + a^2 + 1 = 0).$$

Applying the real automorphs to H_a, we get the new lines

$$(D_a) \quad z = ax + w, y = a^2w;$$
$$(E_a) \quad z = ax + w, y = x + (a + a^2)w;$$

$$(F_a) \quad z = ax + a^2w,\ y = a^2w;$$
$$(L_a) \quad w = ax,\ y = (a^2 + 1)x + a^2z;$$
$$(J_a) \quad z = (a + 1)x + (a^2 + a)w,\ y = x + (a^2 + a)w.$$

Those with the same subscript a are permuted as follows:

$$A: (DF)(LH)(EJ),\ B: (DL)(HE)(FJ),\ C: (DF)(LJ)(EH).$$

Since the 6.3 imaginary lines and the earlier 9 lines give the 27 possible lines, we now have all the lines on (7), a fact checked otherwise.

The automorphs were seen to replace the three lines H_a by all of the 18 imaginary lines. Hence in studying sets of three coplanar lines at least one of which is imaginary, H_a may be taken as one line. The following are seen to give all such sets:

H_a, F_a and $x = y$, $w = z$, in $w + z = a(x + y)$;
H_a, E_a, $y = z = 0$, in $z = ay$;
H_a, D_{a^2}, J_{a^2+a+1}, in $a\,(w + ax) = ay + z$;
H_a, L_a, $x = w = 0$, in $w = ax$;
H_a, J_{a^2}, D_{a^2+a+1}, in $(a^2 + 1)\,(w + ax) = ay + z$.

Those in the second set alone are concurrent, meeting at $(100a)$. Hence there are $18.2/3 = 12$ sets of three imaginary non-concurrent coplanar lines and $18.3/2 = 27$ sets with two imaginary and one real coplanar lines, in 9 of which sets the lines concur (since A, B, C permute the pairs HE, LJ, DF).

Theorem 3. *Just 9 of the 27 straight lines on* (7) *are real. Of the 45 sets of 3 coplanar lines, the lines in 32 sets are not concurrent and those in 13 sets concur.*

5. We consider briefly certain cubic surfaces with singular points. One for which 11 is the only singular point is

$$x^3 + xz^2 + xw^2 + y^2w + yw^2 + xzw = 0. \tag{8}$$

The[7] only real points not on it are 1 and 5. There are only ten lines on this surface, all being real:

$$(a)\ x = y = z,\ (b)\ z = x = y + w,\ (c)\ x = y = z + w,$$
$$(d)\ y = z = x + w,\ (e)\ x = y = 0,\ (f)\ x = w = 0,\ (g)\ x = 0,\ y = w,$$
$$(h)\ w = 0,\ x = z,\ (i)\ x = w,\ y = z,\ (j)\ x = w = y + z.$$

Of these, f and h alone are in $w = 0$, h counting as a double line of intersection with (8). The only sets of three coplanar lines are a, b, h, in $x = z$, meeting at 11; c, d, h, in $z = x + w$, meeting at 11; e, f, g, in $x = 0$, meeting at 3; f, i, j, in $w = x$, meeting at 8; together with the sets of non-concurring lines a, d, i, in $y = z$; a, c, e, in $x = y$; b, c, j, in $y = z + w$; b, d, g, in $y = x + w$.

Theorem 4. *There are only ten lines on (8) and all are real. Three coplanar lines concur if and only if one of them belongs to the singular pair f, h.*

In (8) we replace x by y, z by $z+y$, y by $x+y+ew$, where $e^2+e+1=0$; we obtain the real surface

$$yz(z+w) = w(w^2+x^2+xw), \tag{9}$$

having only ten straight lines of which only two are real. The five real points are those for which $yz = w = 0$, viz., 1, 2, 3, 5, 6.

The only type other than (9) having 1, 2, 3, 5, 6 as its only real points and having a singular point is $w^3+x^2y+xy^2+yz^2+yzw=0$, containing only three straight lines: $y=w=0$; $y=w$, $z=x+bw$, $b^2+b+1=0$; these meet at the only singular point 6.

Another type with only five real points is $xw^2=y^3+yz^2+z^3$, the only lines on which are the six in the planes $x=0$, $w=0$.

[1] And on only the surfaces equivalent to (1) or to $xy(x+y)=0$ under real linear transformation.

[2] They generate all of the 15.6.4.2 real linear automorphs. For, one leaving point 2 fixed must permute the remaining real points 3, 4, 8, 9, 10, 14 in the tangent plane $x=0$ at 2. These six are permuted transitively by (zw) and γ, both of which leave 2 fixed. An automorph which leaves 2 and 3 fixed, and hence the point 8 collinear with them, must permute 4, 9, 10, 14. These are permuted transitively by β and $\gamma\beta(zw)$, which leave 2 and 3 fixed. An automorph leaving 2, 3 and 4 fixed is the identity or $y^1=y+x$, which is the transform of β by $(zw)\pi$.

[3] We readily verify that A, B, C and E_{ab}, D_{abc}, with b and c integers and $a^4=a$, give all of the 27 lines on (1). We have only to consider first the lines whose two equations are solvable for y,z in terms of x,w, and second the lines one of whose equations is $x=0$ or $w=ax$.

[4] If we make also the like replacement on x,y, we change the left member of (4) into $y(x^2+xy+y^2)$. The new surface evidently has its seven real points in $y=w$. Replacing w by $w+y$, x by $x+z$ and then z by $z+y$, we get (4).

[5] Its 16 real linear automorphs are generated by the four: $z^1=z+w$; $x^1=y+w$, $y^1=x+w$; $x^1=y$, $y^1=x$, $z^1=x+y+z$; and $y^1=y+w$, $z^1=y+z$.

[6] They generate all the twelve real linear automorphs of (7). For, such a transformation must leave fixed the pair of points 9, 12 and hence the tangents $x=y$, $x=0$ at those points. These points and planes are interchanged by C. If each is fixed, the transformation is

$$x^1=x,\ y^1=y,\ z^1=by+cz+bw,\ w^1=fy+gz+(f+1)w.$$

Since the lines in $x=0$ are permuted, the case $f=1$, $g=0$, is excluded. For $f=g=0$, we get the identity and A. For $f=0$, $g=1$, we get B, BA. For $f=g=1$, we get AB, BAB.

[7] As shown in the earlier paper, any cubic surface with just 13 real points and having no linear factor is equivalent to (7) or (8).

THE DIFFUSION OF HORSE CULTURE AMONG THE NORTH AMERICAN INDIANS

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK CITY

Presented to the Academy, March 17, 1915

Investigation of the horse-culture complex among the American Indians was undertaken to discover the procedure in a concrete case of culture diffusion, an important anthropological problem of the day. One of the most difficult tasks confronting the anthropologist is the elucidation of the precise complexes by which various traits of culture are produced. Since there is on every hand abundant evidence that many traits of culture are borrowed, or diffused, over large areas, the study of typical concrete instances of diffusion are of the first importance. A number of European anthropologists have been so impressed with the significance of diffusion, that they have developed from it a theory to account for the origin of culture traits. This theory is usually known as that of single origin as opposed to the theory of independent invention. The former asserts that all important traits of culture were invented but once and subsequently gradually diffused; the latter, that the same invention was made independently in many parts of the world, whence its diffusion is but apparent. As everyone knows, the discussion of such problems comes to naught unless concrete cases can be investigated.

The horse-culture complex of the American Indian offers an excellent opportunity to study diffusion, because most of the essential facts are obtainable. The horse was introduced by Europeans at an early date and spread ahead of interior exploration. In particular, many of the tribes west of the Mississippi River became horsemen before their discovery by Europeans. The history of their introduction is briefly outlined by me in the *American Anthropologist*, Vol. 16, No. 1, pp. 1–25. The investigation here reported is the intensive study of collections of riding gear and horse-using appliances to be found in anthropological collections. The material available gives us a representative series for each important tribe in the horse-using area. The detailed comparative study of these specimens has developed many interesting points among the most significant of which are:

1. The Indian has shown no originality. He devised no important

appliances for using horses. He manufactured his own saddles, bridles, etc., but followed precisely a few definite patterns. Though these patterns appear to us as Indian, that is because the English colonists brought with them the English saddle. The Indian model is fundamentally like that of Southern Europe and Asia during the period of American colonization and still survives among the tribes of Patagonia. In general, the complete data will show that the greater part of the horse-complex of the North American Indian was borrowed first by the tribes in contact with the Spanish settlements and then diffused as far as the Plains of Canada without loss or essential modification of detail.

The one striking Indian variation is the habit of mounting on the right side of the horse instead of the left as do Americans and Europeans. The comparative data on this point make it clear that if left to their inclinations right-handed people will mount from the right. Historical data show the European method to have been first introduced into cavalry tactics by Vespasian and to have survived to this day because the sword is worn on the left side. The difference, therefore, is not due to motor differences in the Indian but, like most other culture differences, to historical factors.

2. The Indian did not take the cart. Yet the Spanish colonists rarely if ever used the horse, mule, or donkey for anything but riding and packing; their carts were drawn by cattle. (The great abundance of buffalo no doubt prevented the development of an Indian cattle culture.) On the other hand, the Indian of the Plains had developed dog traction by the travois long before the horse came. When he got the horse, he fitted the travois to him. In any event, it is probable that the established use and simplicity of the travois would have inhibited the use of carts. Thus, while in the travois we have an instance of the use of an Indian invention with the horse, the presence of the horse had nothing to do with its origin.

3. The rapidity and completeness of horse-culture diffusion in America is a good illustration of how fully traits of borrowed culture may be assimilated. In this instance we have sufficient data to determine the general lines of diffusion but such is not often the case. For example, maize culture was once diffused over a large part of North and South America, for the wild plant is found only in one area which must have been the place of origin. In the Old World the spread of horse culture was most likely strictly analogous to its diffusion in America. Returning to our problem, it will be seen how if a non-historical people had

brought Old World horse culture to America, we should be puzzled at the similarities observed between these traits on the two hemispheres, but would probably set it down as another case of assumed independent invention. This investigation shows that the invoking of independent invention, to be more than a plea of ignorance, must rest upon specific data.

The final discussion of this subject will appear in full in the Anthropological Papers of the American Museum of Natural History.

DISCOVERY OF ALGONKIAN BACTERIA

By Charles D. Walcott

SMITHSONIAN INSTITUTION, WASHINGTON

Presented to the Academy, April 9, 1915

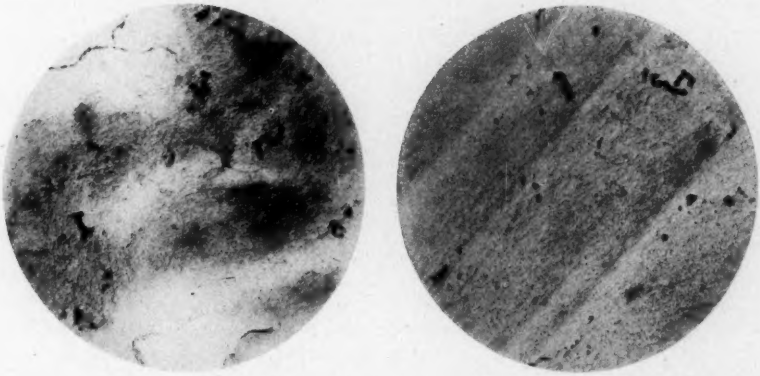
At a meeting of the Botanical Society of Washington, held April 6, 1915, I spoke upon the subject of 'Prepaleozoic Algal Deposits' and in this connection called attention to the existence of bacteria in association with the algal deposits of the Newland limestone, a formation of the Beltian series of Algonkian rocks in central Montana.

Fig. 1. Characteristic groups of *Micrococcus vaccinae*. (After Cohn.) Very highly magnified. [Encyclopædia Britannica, 11th Ed., Vol. III, p. 160, fig. 5–B.]

In a preliminary publication[1] I stated that it was quite probable that bacteria were the most important factor in the deposition of the Algonkian limestones. At that time no definite bacteria had been discovered. From the collections made during the season of 1914 many thin sections were prepared. These were examined by Dr. Albert Mann, plant morphologist of the Department of Agriculture, assisted by Mr. Charles E. Resser of the National Museum, with the result that bacteria were discovered in three of the sections, which were cut from an algal form included under the generic term *Gallatinia* as defined in the preliminary paper upon the Algonkian Algal Flora.[2]

The bacteria consist of individual cells and apparent chains of cells which correspond in their physical appearance with the cells of Micro-

cocci.[3] For the purpose of comparison in this preliminary notice an illustration is here given of a group of recent forms as shown in the *Encyclopedia Britannica*[3] and of the form shown in the cells in the thin sections cut from the fossil alga of the Newland limestone.



Figs. 2 and 3. *Micrococcus* sp. undt. (× about 1100 diameters.) Average size of Micrococci 0.95 to 1.3 microns in diameter. (Slide D.) From locality 401b, Algonkian: Gallatin formation; north side of East Gallatin River, 5 miles (8 km.) east of Logan, Gallatin County, Montana.

[1] Walcott, Pre-Cambrian Algonkian Algal Flora, *Smithsonian Misc. Coll.*, **64**, No. 2 (1914).
[2] Idem, p. 116, pl. 23, fig. 1.
[3] *Encyclopaedia Britannica*, 11th ed., vol. 3, p. 160, fig. 5.

A CORRECTION

(Received March 29, 1915)

In my paper on the earth considered as a heat engine, this volume, page 81, the linear expansion of rock forming minerals was inadvertently taken at 100 times its real value. A square area of superficial rock of relatively low diffusivity would really need to be several hundred degrees hotter than the surrounding areas to be shattered by the compressive stresses called into play by mere difference of temperature. Similarly a mean temperature difference of 40° between oceanic and continental columns overlying the level of isostatic compensation, would by itself account for a difference of level of only about 39 metres.

The error committed does not affect the general argument that the subcontinental shell acts as a heat engine, for it is known that several reversible processes such as elastic strain, expansion, liquefaction and volatilization are

active in this shell, so that the internal heat does not escape to the surface solely by conduction and must therefore do work. Nor is it impossible that after the final consolidation of the superficial rocks, at from 700° to say 1300°, temperature differences of several hundred degrees were set up on equipotential surfaces. It is a matter of certainty that jointing of the rocks is now universal in continental areas and that it has been prevalent from the Archean onwards That jointing impedes the flow of heat, thus exaggerating any primordial inferiority of diffusivity and tending to increase temperature differences, is equally certain. How primeval jointing was initiated is less clear than my mistake made it seem to me, and offers an interesting problem.

GEORGE F. BECKER.